OBJECTIVE: SWBAT identify the number properties by examining equivalent expressions.

Name: ______________________________ Date: ______________

Lesson 1: Number Properties

Do Now:

Simplify the following expressions.

a) 3(8) b) 13+6 c) 7(3) d) 6+13 e) 8(3)

1) Which questions have the same solution? ______________________________

2) What operation(s) did they use? ______________________________

3) Use the order of operations (PEMDAS) to determine if both sides of the equations are equal to each other.

$(-3) \times (8) = (8) \times (-3)$ $\qquad$ $(\frac{1}{2} \times 12) \times 4 = \frac{1}{2} \times (12 \times 4)$

OBJECTIVE: SWBAT identify the number properties by examining equivalent expressions.

Four Properties of Arithmetic:

Property	Addition	Multiplication
Commutative (commute, order)	If a and b are real numbers, then $a+b=b+a$. $1+2=2+1$ $3=3$	If a and b are real numbers, then $a \times b = b \times a$. $6*7=7*6$ $42=42$
Associative (associate, group)	If $a, b,$ and c are real numbers, then $(a+b)+c=a+(b+c)$ $(2+4)+3=2+(4+3)$ $(6)+3=2+(7)$ $9=9$	If $a, b,$ and c are real numbers, then $(ab)c=a(bc)$. $(3*5)8=3(5*8)$ $(15)8=3(40)$ $120=120$

Distributive Property:

Distributive (rainbow, fishing)	If $a, b,$ and c are real numbers, then $a(b+c)=ab+ac$. $2(3+5)=2(3)+2(5)$ $2(8)=6+10$ $16=16$

Practice:

A teacher asked the class to solve the equation $3(x+2)=21$. Robert wrote $3x+6=21$ as his first step. Which property did he use? 1) Associative Property 2) Commutative Property 3) Distributive Property 4) Zero Property of Addition	When solving the equation $4(3x^2+2)-9=8x^2+7$, Emily wrote $4(3x^2+2)=8x^2+16$ as her first step. Which property justifies Emily's first step? 1) Addition Property of Equality 2) Commutative Property of Addition 3) Multiplication Property of Equality 4) Distributive Property of Multiplication Over Addition

OBJECTIVE: SWBAT identify the number properties by examining equivalent expressions.

While solving the equation $4(x+2)=28$, Becca wrote $4x+8=28$. Which property did she use?

1) Distributive
2) Associative
3) Commutative
4) Identity

A method for solving $5(x-2)-2(x-5)=9$ is shown below. Identify the property used to obtain each of the two indicated steps.

$5(x-2)-2(x-5)=9$

(1) $5x-10-2x+10=9$ ________________

(2) $5x-2x-10+10=9$ ________________

$3x+0=9$

$3x=9$

$x=3$

A part of Jennifer's work to solve the equation $2(6x^2-3)=11x^2-x$ is shown below.

Given: $2(6x^2-3)=11x^2-x$
Step 1: $12x^2-6=11x^2-x$

Which property justifies her first step?

1) Identity Property of Multiplication
2) Multiplication Property of Equality
3) Commutative Property of Multiplication
4) Distributive Property of Multiplication Over Subtraction

When solving the equation $12x^2-7x=6-2(x^2-1)$, Evan wrote $12x^2-7x=6-2x^2+2$ as his first step. Which property justifies this step?

1) Subtraction Property of Equality
2) Multiplication Property of Equality
3) Associative Property of Multiplication
4) Distributive Property of Multiplication Over Subtraction

OBJECTIVE: SWBAT identify the number properties by examining equivalent expressions.

Britney is solving a quadratic equation. Her first step is shown below. Problem: $3x^2 - 8 - 10x = 3(2x + 3)$ Step 1: $3x^2 - 10x - 8 = 6x + 9$ Which two properties did Britney use to get to Step 1? I. addition property of equality II. commutative property of addition III. multiplication property of equality IV. distributive property of multiplication over addition 1) I and III 2) I and IV 3) II and III 4) II and IV	When solving $p^2 + 5 = 8p - 7$, Kate wrote $p^2 + 12 = 8p$. What property did Kate use?

Closing Assessment:

When solving for the value of *x* in the equation $4(x - 1) + 3 = 18$, Aaron wrote the following lines on the board.

[line 1]	$4(x-1)+3=18$
[line 2]	$4(x-1)=15$
[line 3]	$4x-1=15$
[line 4]	$4x=16$
[line 5]	$x=4$

Which property was used *incorrectly* when going from line 2 to line 3? Why?

____

____

Standards:
A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

OBJECTIVE: SWBAT determine whether a number is rational or irrational.

Name: ______________________________ Date: ________________

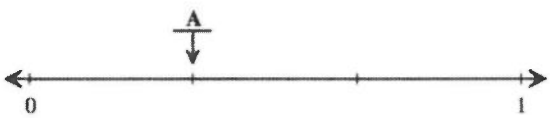
Lesson 2: Types of Numbers

Do Now:

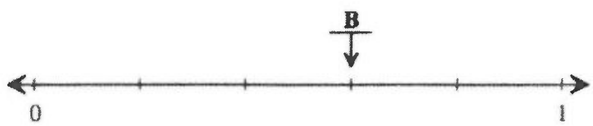
a) On each number line below, label all of the tick marks in fraction form.

b) Using your calculator, give the decimal representation for each lettered tick mark.

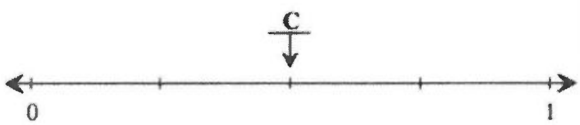
A)



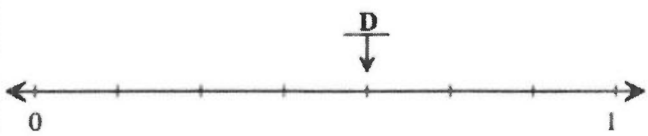
B)



C)



D)



Observations: What do you notice about the numbers at tick marks A, B, C and D?

They are decimals that repeat or stop. We call these rational numbers. Rational numbers can be represented as a fraction or a ratio of two integers.

OBJECTIVE: SWBAT determine whether a number is rational or irrational.

Vocabulary:

Rational Number: a number that can be written as a fraction; a decimal that repeats or stops (terminates)

Irrational Number: a number that can NOT be written as a fraction; a decimal that does not repeat or goes on forever

Reasoning: This is a (ir)rational number because it can (not) be written as a fraction.

Practice:

Determine if the following numbers are rational or irrational. Explain why.

a) 0.36	b) $0.36363636\ldots$	c) $0.\overline{4}$
d) $0.363363336\ldots$	e) $\sqrt{8}$	f) $0.12131415\ldots$
g) $\sqrt{16}$	h) $989989998\ldots$	i) $\pi + 30$

OBJECTIVE: SWBAT determine whether a number is rational or irrational.

Quick Assessment:

Determine if the following numbers are rational or irrational. Explain why.

a) -5.28	b) $0.14141414\ldots$	c) $-\sqrt{5}$
d) $-\pi$	e) $\sqrt{48}$	f) $\sqrt{49}$

Table of Squares:

Squares & Square Roots		
Integers	Squares	Square Roots
1	$1^2 = 1$	$\sqrt{1} = 1$
2	$2^2 = 4$	$\sqrt{4} = 2$
3	$3^2 = 9$	$\sqrt{9} = 3$
4	$4^2 = 16$	$\sqrt{16} = 4$
5	$5^2 = 25$	$\sqrt{2\_} = 5$
6	$6^2 = 36$	$\sqrt{36} = 6$
7	$7^2 = 42$	$\sqrt{49} = 7$
8	$8^2 = 64$	$\sqrt{64} = 8$
9	$9^2 = 81$	$\sqrt{81} = 9$
10	$10^2 = 100$	$\sqrt{100} = 10$
11	$11^2 = 121$	$\sqrt{121} = 11$
12	$12^2 = 144$	$\sqrt{144} = 12$

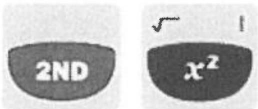
OBJECTIVE: SWBAT determine whether a number is rational or irrational.

Square Roots in the Calculator:

Using the calculator prompts below, find the value of each of the following square and cube roots.

a) $\sqrt{25} =$

How to Type Roots:
For square root, press



b) $\sqrt{64} =$

c) $\sqrt{\frac{4}{9}} =$

Multiplication Table with Highlighted Squares:

MathATube.com

*, /	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	**1**	2	3	4	5	6	7	8	9	10	11	12
2	2	**4**	6	8	10	12	14	16	18	20	22	24
3	3	6	**9**	12	15	18	21	24	27	30	33	36
4	4	8	12	**16**	20	24	28	32	36	40	44	48
5	5	10	15	20	**25**	30	35	40	45	50	55	60
6	6	12	18	24	30	**36**	42	48	54	60	66	72
7	7	14	21	28	35	42	**49**	56	63	70	77	84
8	8	16	24	32	40	48	56	**64**	72	80	88	96
9	9	18	27	36	45	54	63	72	**81**	90	99	108
10	10	20	30	40	50	60	70	80	90	**100**	110	120
11	11	22	33	44	55	66	77	88	99	110	**121**	132
12	12	24	36	48	60	72	84	96	108	120	132	**144**

MathATube.com Multiplication and Division Table

OBJECTIVE: SWBAT determine whether a number is rational or irrational.

Sum and Product of Radicals:

Expression	Solution	Explanation
$\sqrt{9}+\sqrt{4}$	5	Rational; sum of two rational numbers is rational
$\sqrt{4}+\sqrt{2}$	3.414213562	Irrational; the sum of a rational and irrational number is irrational
$\sqrt{3}+\sqrt{2}$ $(2+\sqrt{7})+(-\sqrt{7})$	3.14626437 2	Irrational; the sum of two irrational numbers is irrational or rational*
$\sqrt{9}*\sqrt{4}$	6	Rational; the product of two rational numbers is rational
$\sqrt{4}*\sqrt{2}$	2.828427125...	Irrational; the product of a rational and irrational number is irrational
$\sqrt{3}*\sqrt{2}$ $\sqrt{8}*\sqrt{2}$	2.449489743 16	Irrational; the product of two irrational numbers is irrational or rational*

Practice:

1) Ms. Fox asked her class "Is the sum of 4.2 and $\sqrt{2}$ rational or irrational?" Patrick answered that the sum would be irrational.

   State whether Patrick is correct or incorrect. Justify your reasoning.

OBJECTIVE: SWBAT determine whether a number is rational or irrational.

2) Given: $L = \sqrt{2}$
$M = 3\sqrt{3}$
$N = \sqrt{16}$
$P = \sqrt{9}$

Which expression results in a rational number?

(a) $L + M$ (b) $N + P$

(c) $M + N$ (d) $P + L$

3) Jakob is working on his math homework. He decides that the sum of the expression $\frac{1}{3} + \frac{6\sqrt{5}}{7}$ must be rational because it is a fraction. Is Jakob correct? Explain your reasoning.

4) For which value of P and W is $P + W$ a rational number?

(a) $P = \frac{1}{\sqrt{3}}$ and $W = \frac{1}{\sqrt{6}}$

(b) $P = \frac{1}{\sqrt{4}}$ and $W = \frac{1}{\sqrt{9}}$

(c) $P = \frac{1}{\sqrt{6}}$ and $W = \frac{1}{\sqrt{10}}$

(d) $P = \frac{1}{\sqrt{25}}$ and $W = \frac{1}{\sqrt{2}}$

OBJECTIVE: SWBAT determine whether a number is rational or irrational.

5) State whether $7 - \sqrt{2}$ is rational or irrational. Explain your answer.

6) A teacher wrote the following set of numbers on the board:

$$a = \sqrt{20} \qquad b = 2.5 \qquad c = \sqrt{225}$$

Explain why $a + b$ is irrational, but $b + c$ is rational.

7) Is the sum of $3\sqrt{2}$ and $4\sqrt{2}$ rational or irrational? Explain your answer.

OBJECTIVE: SWBAT determine whether a number is rational or irrational.

8) Given the following expressions:

I. $-\frac{5}{8}+\frac{3}{5}$ III. $(\sqrt{5})\cdot(\sqrt{5})$

II. $\frac{1}{2}+\sqrt{2}$ IV. $3\cdot(\sqrt{49})$

Which expression(s) result in an irrational number?

(a) II, only (b) I, III, IV

(c) III, only (d) II, III, IV

Closing Assessment:

Determine if the product of $3\sqrt{2}$ and $8\sqrt{18}$ is rational or irrational. Explain your answer.

Standards: CCSS.Math.Content.N-Q.A.2: Define appropriate quantities for the purpose of descriptive modeling. CCSS.Math.Content.A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context. N.RN.B- Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

OBJECTIVE: SWABT analyze a set of data for its measures of central tendency.

Name: ______________________________ Date: ________________

Lesson 3: Measures of Central Tendency

Vocabulary:

Data: a set of numbers that have been collected for study

Mean: average from a set of data; sum of data divided by the number of data

Median: middle number of a set of data; must arrange in numeric order (least to greatest)

Mode: number that appears most often in a set of data

Range: difference between the highest and lowest number in the data set

Example:

Mike looked at his quiz scores shown below for the first semester of his Algebra class.
Semester 1: 78, 91, 88, 83, 94

Determine the mean, median, mode, and range.

Mean: $78 + 91 + 88 + 83 + 94 = 434 \div 5 = 86.8$

Median: 78, 83, 88, 91, 94

Mode: none

Range: $94 - 78 = 16$

OBJECTIVE: SWABT analyze a set of data for its measures of central tendency.

Video Lesson

Scan the QR Code to watch a Khan Academy video introducing the measures of central tendency defined. Follow along with the following problem.

Data: 4 3 1 6 1 7

OBJECTIVE: SWABT analyze a set of data for its measures of central tendency.

Practice:

1) Liz collects data from two different companies, each with four employees. The results of the study, based on each worker's age and salary, are listed in the tables below.

Company 1

Worker's Age in Years	Salary in Dollars
25	30,000
27	32,000
28	35,000
33	38,000

Company 2

Worker's Age in Years	Salary in Dollars
25	29,000
28	35,500
29	37,000
31	65,000

Which statement is true about these data?

a) The median salaries in both companies are greater than $37,000.
b) The mean salary in company 1 is greater than the mean salary in company 2.
c) The salary range in company 2 is greater than the salary range in company 1.
d) The mean age of workers at company 1 is greater than the mean age of workers at company 2.

2) The table below shows the annual salaries for the 24 members of a professional sports team in terms of millions of dollar.

0.5	0.5	0.6	0.7	0.75	0.8
1.0	1.0	1.1	1.25	1.3	1.4
1.4	1.8	2.5	3.7	3.8	4
4.2	4.6	5.1	6	6.3	7.2

The team signs an additional player to a contract worth 10 million dollars per year. Which statement about the median and mean is true?

a) Both will increase.
b) Only the median will increase.
c) Only the mean will increase.
d) Neither will change.

OBJECTIVE: SWABT analyze a set of data for its measures of central tendency.

3) The two sets of data below represent the number of runs scored by two different youth baseball teams over the course of a season.

Team A: 4, 8, 5, 12, 3, 9, 5, 2

Team B: 5, 9, 11, 4, 6, 11, 2, 7

Which set of statements about the mean and median is true?

a) mean A < mean B
median A > median B

b) mean A > mean B
median A < median B

c) mean A < mean B
median A < median B

d) mean A > mean B
median A > median B

Closing Assessment:

Which of the following is true about the data set {3, 5, 5, 7, 9}?

a) median > range

b) median = mean

c) mean > median

d) median > mean

Standards: CC.S.ID.1: Represent data with plots on the real number line (dot plots, histograms, and box plots).

OBJECTIVE: SWBAT interpret and create graphical representations from a given set of data.

Name: ______________________ Date: ____________

Lesson 4: Graphical Representations of Data

Statistical Summary:

Separating data into subsets is a useful way to summarize and compare data sets.

Quartiles are values that divide a data set into four equal parts.

The median also known as the second quartile or Q_2, separates the data into upper and lower halves.

The **first quartile**(Q_1) is the median of the lower half of the data.

The **third quartile**(Q_3) is the median of the upper half of the data.

The interquartile range is the difference between the third and first quartiles.

****WARNING! For a data set that has an odd number of values, do not include the median in either half when finding the first and third quartiles. ****

Example:
Determine the statistical summary for the data below.

125, 80, 140, 135, 126, 140, 350, 75

On Calculator:
STAT → EDIT
Enter data into L1
STAT → CALC → (1) 1-Var Stats
***Be sure L1is being used.**

Step 1: Arrange the data in order from least to greatest.

Step 2: Find the minimum, maximum, and median.

Minimum: ________ Median: _______

Maximum: _________

Step 3: Find the first and third quartile.

First Quartile: ________ Third Quartile: ________

Calculator - Statistics Key

$\bar{x}$ = mean
$\sum x$ = sum of the data
$\sum x^2$ = sum of squares of the data
Sx = sample standard deviation
σx = population standard deviatio
n = sample size (# of pieces of
min X = smallest data entry
Q_1 = first quartile
med = median (second quartile)
Q_3 = third quartile
max X = largest data entry

OBJECTIVE: SWBAT interpret and create graphical representations from a given set of data.

Practice:

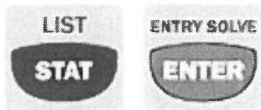
Using the calculator, find the statistical summary of each data set.

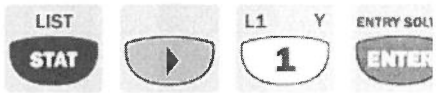
a) 95, 85, 75, 85, 65, 60, 100, 105, 75, 85, 75

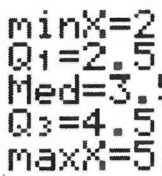
Minimum: ________ Maximum: ________

Median: ________

First Quartile: ________ Third Quartile: ________

Using the Calculator:
Press


Type data into L_1
Press


Press Enter until you get numbers.
Scroll down until you see:


$\bar{x}$= mean

Minimum: lowest value in data

Q1 → Quartile 1 → 25th Percentile: middle of the lower half

Median → Quartile 2 → 50th Percentile: middle of all data, divides data in half

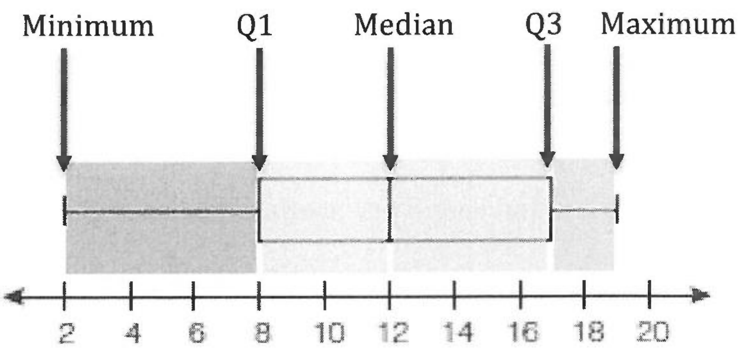
Q3 → Quartile 3 → 75th Percentile: middle of the upper half

Maximum: highest value in data

Box Plots:

A box plot is a graph that summarizes a set of data by displaying it along a number line. It consists of three parts: a box and two lines. Also referred to as a box-and-whiskers plot.

Example:



OBJECTIVE: SWBAT interpret and create graphical representations from a given set of data.

Practice:

Let's construct a box plot for the following set of data.

The Data: Math test scores in Algebra class: 80, 75, 90, 95, 65, 65, 80, 85, 70, 100

Step 1: Arrange the data in order from least to greatest.

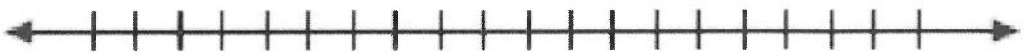
Step 2: Identify the minimum, maximum, and median.

Minimum: ________ Maximum: _________ Median: ________

Step 3: Find the first and third quartile.

First Quartile: ________ Third Quartile: ________

Step 4: Place a circle above each of these values in relation to their location on an equally spaced number line.



Step 5: Draw a box with ends through the points for the first and third quartiles. Then draw a vertical line through the box at the median point. Now, lines (or whiskers) from each end of the box to these minimum and maximum values.

Extension: Which statistic can *not* be determined from the box plot representing the scores on the mast test in Algebra class?

a) the lowest score
b) the median score
c) the highest score
d) the score that occurs most frequently

OBJECTIVE: SWBAT interpret and create graphical representations from a given set of data.

Application:

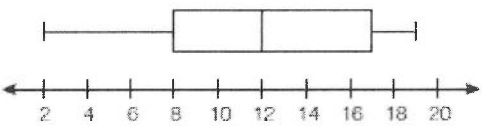
Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.

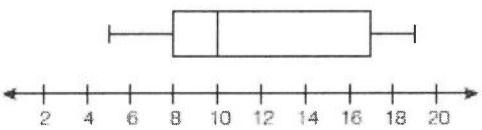
	Sun	Mon	Tues	Wed	Thurs
Week 1	4	3	3.5	2	2
Week 2	4.5	5	2.5	3	1.5
Week 3	4	3	1	1.5	2.5

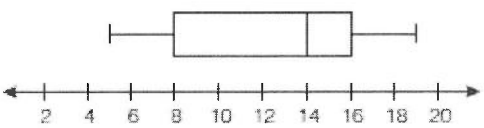
Using an appropriate scale on the number line below, construct a box plot for the 15 values.

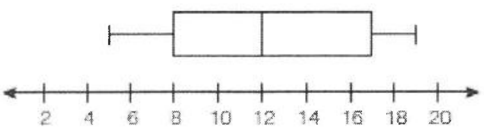
Closing Assessment:

The data set 5, 6, 7, 8, 9, 9, 9, 10, 12, 14, 17, 17, 18, 19, 19 represents the number of hours spent on the Internet in a week by students in a mathematics class. Which box plot represents the data?

1)


2)


3)


4)


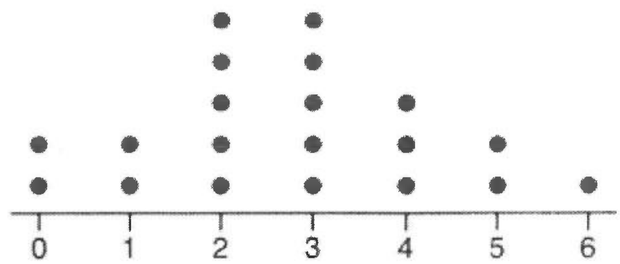
Explain your answer choice:

OBJECTIVE: SWBAT interpret and create graphical representations from a given set of data.

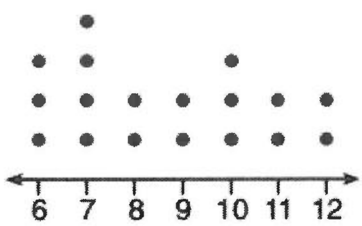
Dot Plots:

A dot plot is a graphical display of data using dots on a number line where each dot represents a response. The image could be compared to a bar graph.

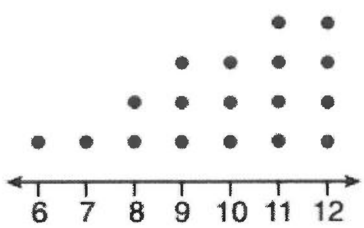
1) The dot plot shown below represents the number of pets owned by students in a class.
Which statement about the data is *not* true?
   a) The median is 3.
   b) The interquartile range is 2.
   c) The mean is 3.
   d) The data contain no outliers.



2) Noah conducted a survey on sports participation. He created the following two dot plots to represent the number of students participating, by age, in soccer and basketball.



Soccer Players' Ages



Basketball Players' Ages

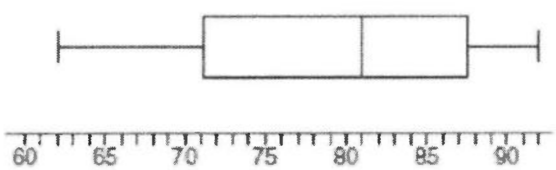
Which statement about the given data sets is correct?
   a) The data for soccer players are skewed right.
   b) The data for soccer players have less spread than the data for basketball players.
   c) The data for basketball players have the same median as the data for soccer players.
   d) The data for basketball players have a greater mean than the data for soccer players.

OBJECTIVE: SWBAT interpret and create graphical representations from a given set of data.

Standards: CC.S.ID.1: Represent data with plots on the real number line (dot plots, histograms, and box plots).

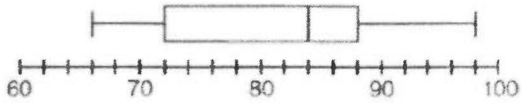
Enrichment:

The accompanying diagram shows a box-and-whisker plot of student test scores on last year's Mathematics A midterm examination.



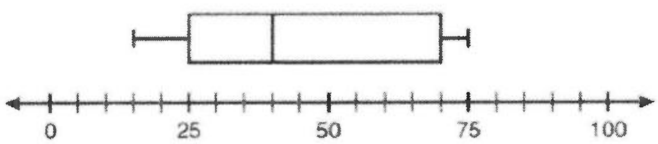
What is the median score?
1) 62
2) 71
3) 81
4) 92

The box-and-whisker plot below represents the math test scores of 20 students.



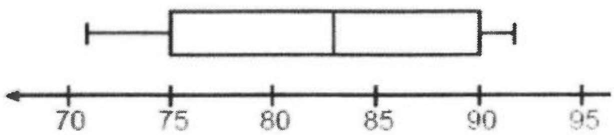
What percentage of the test scores are *less than* 72?
1) 25
2) 50
3) 75
4) 100

What is the range of the data represented in the box-and-whisker plot shown below?



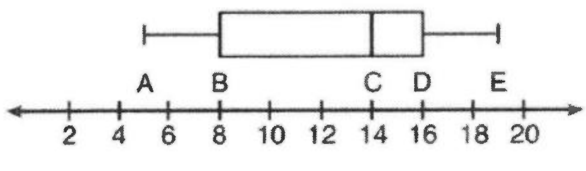
1) 40
2) 45
3) 60
4) 100

The box plot below summarizes the data for th average monthly high temperatures in degrees Fahrenheit for Orlando, Florida.



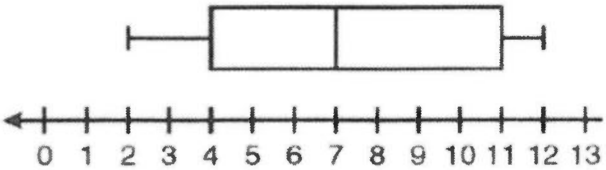
The third quartile is
1) 92
2) 90
3) 83
4) 71

The box-and-whisker plot shown below represents the number of magazine subscriptions sold by members of a club.



Which statistical measures do points *B*, *D*, and *E* represent, respectively?
1) minimum, median, maximum
2) first quartile, median, third quartile
3) first quartile, third quartile, maximum

Based on the box-and-whisker plot below, whic statement is *false*?



1) The median is 7.
2) The range is 12.
3) The first quartile is 4.
4) The third quartile is 11.

OBJECTIVE: SWBAT add and subtract polynomials by identifying like terms.

Name: ______________________________ Date: ______________

Lesson 5: Operations with Polynomials

What is a Term?

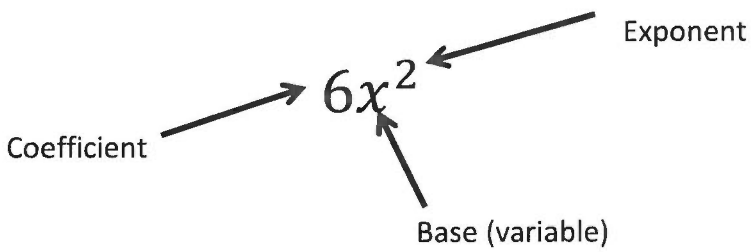
Monomial- an expression consisting of one term

Coefficient- number before variable

Base- number being multiplied (variable)

Exponent- how many times base is multiplied

Variable- a value that changes depending on a problem, letters represent variables



Like Terms:

Monomials are exactly the same except for their coefficients.

Same variable raised to the same exponent.

Which sets have like terms? If they are not like terms, state why.

$7x$	$7x$	
$4x^2$	$3x$	
$4y$	$3y$	
$5y$	$3xy$	

OBJECTIVE: SWBAT add and subtract polynomials by identifying like terms.

Combine Like Terms (CLT):

1) Identify like terms.
2) Add the coefficients. Do NOT change the variables.
   a. If the signs are the same, add the terms and keep the sign.
   b. If the signs are different, subtract the terms and take the sign of the larger number.

Examples:

A) $5x + 3x = 8x$

B) $-2z - 9z = -11z$

C) $-4y + 8y = 4y$

Practice:

$2x^2 - x^2$	$8m^4 - (-3m^4)$
$-4x + 3y$	A rectangular football field has a length of $6z^3$ and a width of $9z^3$. Find the perimeter.

OBJECTIVE: SWBAT add and subtract polynomials by identifying like terms.

Quick Assessment:

$5xy + 7xy$	$9p^2 - 12p^2$	$-10g - 6g$

Combining Polynomials:

1. Distribute the number or sign outside of the parenthesis.
2. Determine like terms by circling them.
3. Add or subtract the coefficients.
4. Leave the exponents alone.

Directions: Find each sum or difference by combining the parts that are alike.

a. $(4x^2 + x + 7) + (2x^2 + 3x + 1)$

$$4x^2 + x + 7 + 2x^2 + 3x + 1$$
$$4x^2 + 2x^2 + x + 3x + 7 + 1$$
$$6x^2 + 4x + 8$$

b. $(3x^3 - x^2 + 8) - (x^3 + 5x^2 + 4x - 7)$

c. What is the difference when $3x^2 - 8x$ is subtracted from $2x^2 + 3x$?

OBJECTIVE: SWBAT add and subtract polynomials by identifying like terms.

Applications:

1) Determine the perimeter of a triangle whose sides are $2x^2 + 3x$, $5x^2 - 1$, and $3x^2 + 7x - 1$.

2) Subtract $3x^2 - 8x + 6$ from $4x^2 + 2x - 9$.

3) In the year 2001, the average cost of a bicycle could be modeled by the equation $C = -5x^2 + 600$ where x is the number of years since 2001. By the year 2015 the average cost had changed, so it could be modeled by the equation $C = -10x^2 + 800$. Find the difference in the average costs for a bicycle between 2015 and 2001.

OBJECTIVE: SWBAT add and subtract polynomials by identifying like terms.

Closing Assessment:

Find the difference: What is the result when $4x^2 + 7x - 5$ is subtracted from $9x^2 - 2x + 3$?

Practice: Simplify each of the following polynomials

a) $(5x^2 - 10x + 1) + (-2x^2 - 10x + 3)$	**b)** $(4x^2 + x) - (2x^2 - 3x + 3)$
c) $(11x^2 - 9x + 2) - (x^2 - 7x)$	**d)** $(10x^2 - 2) + (9x - 2)$
e) $(-9) - (x^2 + 9x + 9)$	**f)** $(-3x^2 - 11x + 2) - (-2x^2 - 9x + 3)$

OBJECTIVE: SWBAT add and subtract polynomials by identifying like terms.

Directions: Circle the operation used for each problem by identifying key words. Then solve each problem.

Add **Subtract** **1)** What is the sum of $4x^2 - 10x - 1$ and $8x^2 + 2x + 4$	**Add** **Subtract** **2)** What is $-6x^2 - 9x - 2$ subtracted from $3x^2 + 8x - 9$?
Add **Subtract** **3)** What is $6x^2 - 9$ less than $10x^2 + 8x - 3$?	**Add** **Subtract** **4)** What is $5x^2 + 9x - 11$ and $-2x^2 - 3$ in total?

Standards: A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

Name: ______________________________ Date: ______________

Lesson 6: Multiplying Polynomials

Law of Exponents:

Multiplying Powers with the Same Base:

$$a^m * a^n = a^{m+n}$$

Examples: $x^2(x^3) = x^{2+3} = x^5$

$2z^4(5z^5) = 2 \cdot 5z^{4+5} = 10z^9$

Ask Yourself:

What's the base of each term?

Are they the same?

Then, add the exponents.

How to Multiply Monomials:

1) Multiply the coefficients.

2) If like bases, then keep the base and add the exponents.

Examples:

Find the products of the following monomials.

$-2x^4(7x^3)$ $-2 \cdot 7x^{4+3} = -14x^7$	$5x^2y^6(3x^8y)$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

Practice:

Find the products of the following monomials.

$8x^5(3x)$
$-5x^3(3x^7)$
$10x^2y^5(9x^3y)$
$-8x^6y^{10}(-5x^6y^8)$
$9x^2y^9(8x^{10}y^9)$

$-90x^5(5x^5)$
$5x^4(3x^9)$
$-100x^2y^4(5x^{10}y^9)$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

$10x^{20}y(x^3y^{90})$
$-11x^2(11x^6)$

Application:

A rectangular garden has a length of $8x^3y$ and a width of $6x^4y^5$. Find the area of the garden.

Quick Assessment:

Find the products of the following monomials.

$3x^2(x^6)$	$-7x^3y^4(5x)$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

How to Multiply Polynomials by Monomials:

1) Distribute any outside terms to the ones inside the parentheses.

2) Multiply the coefficients.

3) If like bases, then keep the base and add the exponents.

Examples:

1) $-5x(x^2 - 2x + 4)$

$-5x(x^2) - 5x(-2x) - 5x(4)$

$-5 \cdot 1x^{1+2} - 5 \cdot -2x^{1+1} - 5 \cdot 4x$

$-5x^3 + 10x^2 - 20x$

2) $-6x^3(-7x^5 + 3)$

$-6x^3(-7x^5) - 6x^3(3)$

$42x^8 - 18x^3$

Practice:

Use the distributive property to multiply the monomial by the polynomial.

A) $2x(4x^2 + 3x)$

B) $8x^5(-2x^2 + 4x - 9)$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

1) $(2r^2 - 5)\,3r$

2) $-3x^2y\,(5xy^2 + xy)$

3) Simplify the expression: $3(x-2) - 2(x-1)$

4) $3a(4+a)$

5) $4x(x^3 - 10)$

6) $(-5w - 3)w^2$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

Multiplying Binomials:

Box Method:

1. Create a 2-by-2 box.
2. Write a binomial on the top and side of the box.
3. Multiply the terms that line up for each box.
4. Write all of the terms as an expression.
5. Combine like terms.

Example:

$$(x + 2)(x + 3)$$

x	$+2$	
x^2	$+2x$	x
$+3x$	$+6$	$+3$

$$x^2 + 2x + 3x + 6$$

$$x^2 + 5x + 6$$

Distributive Method:

1. Circle the first term in the first set of parentheses.
2. Multiply this term with both terms in the second set of parentheses.
3. Repeat steps 1 and 2 for the second term in the first set of parentheses.
4. Combine like terms.

Example:

$$(x + 4)(x + 5)$$

$$x(x) + x(5) + 4(x) + 4(5)$$

$$x^2 + 5x + 4x + 20$$

$$x^2 + 9x + 20$$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

Independent Practice:

Multiply the binomials through the box method or the distributive property.

1) $(x+6)(x-6)$	2) $(x+3)^2$
3) $(x-3)(x+6)$	4) $(2x-1)(3x^2+4)$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

Multiplying Polynomials:

Box Method:

6. Count the number of terms in the first and second set of parentheses.
7. Create a box with the number of terms from the parentheses.
8. Write the polynomials along the top and side.
9. Multiply the terms that line up for each box.
10. Write all of the terms as an expression.
11. Combine like terms.

Example:

$$(x + 4)(x^2 - 3x + 1)$$

x	$+4$	
x^3	$+4x^2$	x^2
$-3x^2$	$-12x$	$-3x$
$+x$	$+4$	$+1$

$$x^3 - 3x^2 + x + 4x^2 - 12x + 4$$

$$x^3 + x^2 - 11x + 4$$

Distributive Method:

5. Circle the first term in the first set of parentheses.
6. Multiply this term with all of the terms in the second set of parentheses.
7. Repeat steps 1 and 2 for each term in the first set of parentheses.
8. Combine like terms.

Example:

$$(3x^3 + 6x + 2)(2x^2 - 5)$$

$$3x^3(2x^2) + 3x^3(-5) + 6x(2x^2) + 6x(-5) + 2(2x^2) + 2(-5)$$

$$6x^5 - 15x^3 + 12x^3 - 30x + 4x^2 - 10$$

$$6x^5 - 3x^3 + 4x^2 - 30x - 10$$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

Practice:

Multiply the binomials through the box method or the distributive property.

1) $(x - 1)(x^3 + 6x^2 - 5)$	2) $(x - 2)^3$
3) Find the product of $(11x^2 + 9x + 3)$ and $(3x^2 - 4)$ written in standard form	4) $(2x^2 + 10x + 1)(x^2 + x + 1)$

OBJECTIVE: SWBAT multiply polynomials by applying laws of exponents.

5) $(7x^2 + 2x - 5)(2x^3 + 4)$	6) $(x^3 + 4x - 2)(2x + 3)$

7) Find the product of $(5x^2 + 4x)$ and $(-10x^2 + 4x + 2)$ written in standard form.

Standards: HSA-APR.A.1: Perform arithmetic operations on polynomials (i.e. add, subtract, multiply, and divide polynomials).

OBJECTIVE: SWBAT create an expression or equation by identifying the operation that corresponds with a word or phrase.

Name: ______________________________ Date: ______________

Lesson 7: Expressions & Equations

Translating Terms:

The following words and phrases relate to different operations as shown below.

Add	Subtract
More than Sum Total Increased by In all Plus	Difference Fewer Decreased by Minus Less than* From* Take away Reduced by *switches the order
Multiply	**Divide**
Twice (x2) Double (x2) Triple (x3) Of Product Times Every Each Per ←	Quotient Divided by Divided equally Half (÷ 2) → Per

OBJECTIVE: SWBAT create an expression or equation by identifying the operation that corresponds with a word or phrase.

Expressions:

Expressions consists of:

- variables
- numbers
- operations

They do NOT have an equal = sign.

Examples:
- 10
- 2x
- 7 + y
- 5z – 6
- v
- $x^2 + 4x - 8$

Translating Sentences:

1) Identify the numbers and the unknown values, this will be 'x.'

2) Identify the operation(s) $(+, -, \times, or\ \div)$.

3) Use the operation(s) to connect the numbers and unknown values.

Practice:

Write the algebraic expression by identifying the operations.

1) Half of a number added to 15 $\frac{1}{2}$ x + 15 $\frac{1}{2}x + 15$	2) The product of x and 29
3) The sum of 5 and c subtracted from 9	4) The quotient of $2x$ and 8

OBJECTIVE: SWBAT create an expression or equation by identifying the operation that corresponds with a word or phrase.

Equations:

Equations are two expressions that are set equal to each other.

They DO have an equal = sign.

Examples:

- $x = 8$
- $y - 5 = 3$
- $4z + 2 = 14$
- $x^2 + 5x - 6 = 0$

Translating Statements:

1. **Identify** the unknown (what you are looking for) and assign a **variable** to it (i.e **"each", "per", "a number"**).

2. Look for key words such as **"is", "is the same as", "is identical to", "equal to", "total", "equivalent"** to indicate where the equal sign should be placed.

3. Write the sentence as an equation.

Example: Translate the following sentence into an equation. The boxes indicate where you need to place operations (+, −,×, and ÷) and the equal sign.

Ten less than a number is 14.

$10 \quad - \quad x \quad = 14$

*Less than switches order!

$x - 10 = 14$

Three is ten more than five times a number. ____________________

OBJECTIVE: SWBAT create an expression or equation by identifying the operation that corresponds with a word or phrase.

Practice:

Write the algebraic expression by identifying the operations.

1) Fifty decreased by a number is twenty.	2) Twice a number increased by 14
3) The product of 7 and a number subtracted from 20	4) Seven less than a number is twelve.
5) 8 more than 3 times a number is 29.	6) Twenty less than a number
7) Three times a number divided by 6	8) Twice a number is ten more than a number.
9) Half of a number minus 5 is the sum of k and 13.	10) 4 less than the product of 5 and a number.
11) One less than the quotient of 4 and y.	12) Fifteen times a number subtracted from 80 is 25.

OBJECTIVE: SWBAT create an expression or equation by identifying the operation that corresponds with a word or phrase.

Quick Assessment:

Match each expression with its corresponding statement.

______1) $-4n = -6$	a) Three times a number increased by two
______2) $2n + 1$	b) Five less than a number is 10
______3) $n - 5 = 10$	c) Two times the quantity of one less than a number
______4) $3n + 2$	d) Negative four times a number is -6
______5) $2(n - 1)$	e) The sum of two times a number and one

Evaluating Functions:

<table>
<tr>
<td>
Evaluating an expression is the same as finding the range value.
1. Substitute x into the function.
2. Evaluate.
3. $f(x)$ equals the range value (the y).

Example:
Using the function $f(x) = -3x + 1$.
Find $f(2)$.
$x = 2$
$f(2) = -3(2) + 1$
$f(2) = -6 + 1$
$f(2) = -5$
</td>
<td>
Working Backwards:
1. Given y = # or $f(x)$ = #.
2. Substitute the number for y or $f(x)$.
3. Solve for x.

Example:
Using the function $f(x) = -3x + 1$.
Find x when $f(x) = 10$.
$f(x) = 10 = y$
$10 = -3x + 1$
$9 = -3x$
$-3 = x$
</td>
</tr>
</table>

OBJECTIVE: SWBAT create an expression or equation by identifying the operation that corresponds with a word or phrase.

Practice:

A) If f(x)=25, then find x when f(x)=4x-3.	B) The value in dollars, $v(x)$, of a certain car after x years is represented by the equation $v(x) = 25{,}000(0.86)^x$. To the nearest dollar, how much more is the car worth after 2 years than after 3 years?

C) If $f(x) = \sqrt{25 - x^2}$, find $f(3)$.	D) A population of wolves in a county is represented by the equation $P(t) = 80(0.98)^t$, where t is the number of years since 2006. Predict the number of wolves in the population in the year 2016.

Quick Assessment:

1) If $g(x) = 4x + 7$, find $g(4)$.

2) If $f(x) = 3x - 4$ and $g(x) = x^2$, find the value of $f(3) - g(2)$.

Standards: Standards: .6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers. CC.F.IF.2: Functional Notation: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

OBJECTIVE: SWBAT identify the inverse operation to apply in order to solve an equation.

Name: ______________________________ Date: __________________

Lesson 8: Solving Equations

Inverse Operations:

Inverse operations are opposite operations.

Operation	Inverse Operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication
Square^2	Square Root $\sqrt{\ }$
Square Root $\sqrt{\ }$	Square^2

How to Solve Equations:

Solve the equation: $2x - 12 = 30$ $+12 \quad +12$ $2x = 42$ $\div 2 \quad \div 2$ $x = 21$ Check: $2(21) - 12 = 30$ $30 = 30$	**How to Solve Equations:** When solving equations, we are finding the value of the variable. ****GOAL: You want to isolate the variable.** Step 1: Draw a line to **separate** both sides of the equation. Step 2: Identify the variable. Step 3: Identify the operation used. Step 4: Apply the inverse operation to **BOTH** sides of the equal sign. Step 5: Write your answer. Step 6: Check your work by substituting your answer for the variable in the original equation and solve.

OBJECTIVE: SWBAT identify the inverse operation to apply in order to solve an equation.

Practice:

Solve and check the following two-step equations.

#	Solve	Check
1)	$2n + 10 = -2$	
2)	$-15 = 4m + 5$	
3)	$-2 = 2 + \frac{x}{4}$	
4)	$-3x + 7 = 16$	

OBJECTIVE: SWBAT identify the inverse operation to apply in order to solve an equation.

Quick Assessment:

Solve and check the following two-step equation: $\mathbf{4x + 15 = 75}$

Substitution:

After solving an equation, we replace the variable with our solution to check our work. If you are given the answer, you may be asked to evaluate, which is the same action.

Example:

Check that $x = 2$ is the solution to the equation $3x - 7 = -1$.

$$3(2) - 7$$
$$6 - 7$$
$$-1$$

How to Solve Multi-Step Equations:

Solve the equation:	**How to Solve Equations:**
$5x + 7x + 6 = 78$ $12x + 6 = 78$ $-6 \quad -6$ $12x = 72$ $\div 12 \quad \div 12$ $x = 6$	Step 1: Draw a line to **separate** both sides of the equation. Step 2: Distribute if necessary. Step 3: Combine like terms. Step 4: Add or subtract the constants. Step 5: Multiply or divide the term with the variable. Step 6: Check your work.

OBJECTIVE: SWBAT identify the inverse operation to apply in order to solve an equation.

Practice:

Solve and check the following multi-step equations.

1) $3(x + 7) = 26$	2) $\frac{x + 6}{4} - 6 = 10$
3) $3.5x - 0.02x = 1.24$	4) $4(x - 9) + x = 64$

Quick Assessment:

Solve and check the following multi-step equation: $7x + 4 - 3 = 15$

OBJECTIVE: SWBAT identify the inverse operation to apply in order to solve an equation.

How to Solve Multi-Step Equations:

Solve the equation:	**How to Solve Equations:**
$5 - 2x = -4x - 7$ $-5 \qquad\qquad -5$ $-2x = -4x - 12$ $+4x \quad +4x$ $2x = -12$ $\div 2 \quad \div 2$ $x = -6$	Step 1: Draw a line to **separate** both sides of the equation. Step 2: Simplify both sides. Step 3: Move variable terms to the left. Step 4: Add or subtract the constants. Step 5: Multiply or divide the term with the variable. Step 6: Check your work.

Practice:

Solve and check the following multi-step equations.

1)	2)
$\frac{7}{3}\left(x + \frac{9}{28}\right) = 20$	$6(x - 2) = 36 - 10x$

OBJECTIVE: SWBAT identify the inverse operation to apply in order to solve an equation.

3) $2x - (x + 1) = 0$	4) $3(x + 1) - 5x = 12 - (6x - 7)$
6) $\frac{3}{5}(x + 2) = x - 4$	5) $13x - 2(x + 4) = 8x + 1$

Standards: HSA.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letter

OBJECTIVE: SWBAT determine when an algebraic fraction is undefined and its solution set.

Name: ______________________________ Date: ________________

Lesson 9: Unique Equations

How to Solve with Variables in the Denominator:

1) Set the denominator not equal to zero ($\neq 0$).

2) Solve for the variable.

3) In the original equation, cross multiply.

   Note: If one side is not a fraction, put it over 1.

4) Use inverse operations to solve for the variable.

5) Write the solution set (what x is equal to) from least to greatest in brackets, { }.

Example:

Consider the equation: $\frac{1}{x} = \frac{3}{x-2}$

a. Determine when the equation is undefined.	b. Solve the equation for x.
$x \neq 0$ $x - 2 \neq 0$ $+2 \quad +2$ $x \neq 2$	$\frac{1}{x} = \frac{3}{x-2}$ $3x = x - 2$ $-x \quad -x$ $2x = -2$ $\div 2 \quad \div 2$ $x = -1$
c. Write the solution set. **If the solution in step b is the same number as the undefined x in part a, then there is a contradiction and there is no solution or an empty set { } $\{x \mid x = -1; x \neq 0, x \neq 2\}$	

OBJECTIVE: SWBAT determine when an algebraic fraction is undefined and its solution set.

Practice:

Determine when each equation is undefined. Then, solve the equation for x.

A) $\frac{5}{x} = 1$	B) $\frac{x}{x+1} = 4$
C) $\frac{2}{x} = \frac{3}{x-4}$	D) $\frac{x}{x+6} = -\frac{6}{x+6}$
E) $\frac{x-3}{x+2} = 0$	F) $\frac{x+3}{x+3} = 5$

OBJECTIVE: SWBAT determine when an algebraic fraction is undefined and its solution set.

Vocabulary:

- A ***number sentence*** is a statement of equality between two numerical expressions (do not contain variables). Example: $4 + 12 = 8 * 2$
- An ***algebraic equation*** is a statement of equality between two expressions (often contains variables). Example: $2(x + 12) = -16$
- An algebraic equation is **true** if both expressions are equivalent (left side equals the right side of the equal sign).
- If the expressions are **NOT** equivalent we call the equation false. *True* and *false* are called truth-values.

Solutions to Equations:

A value for a variable is called a solution to the equation if, when substituted into both expressions, results in the equation being true. We find the solution(s) by solving the equation.

An **algebraic equation** is a statement of equality between two quantities.

Most algebraic equations are TRUE when certain values are substituted for the variable and are FALSE for all other values. The values that make the equation TRUE are called **solutions.** There are unique equations that are always TRUE or always FALSE, no matter what values are substituted.

Equations that are TRUE under certain conditions:

When we get a solution of a $variable = number$ (ex: $x = 2$)

The equation has one solution

Equations that are ALWAYS TRUE:

When we get a solution of $number = the\ same\ number$ (ex: $5 = 5\ or\ x + 1 = x + 1$)

The equation has many solutions.

Equations that are ALWAYS FALSE:

When we get a solution of $number = different\ number$ (ex: $4 = 3$)

The equation has no solutions.

OBJECTIVE: SWBAT determine when an algebraic fraction is undefined and its solution set.

Exploratory Exercise #1:

Find the solution to the following equation: $x + 10 = 2x + 3$.

$$\begin{aligned} x + 10 &= 2x + 3 \\ -10 \quad & \quad -10 \\ x &= 2x - 7 \\ -2x \quad & -2x \\ -x &= -7 \\ \div -1 \quad & \div -1 \\ x &= 7 \end{aligned}$$

There are no other values for x that would make this equation true.

Sum It Up #1:

If there is only one value of the variable that makes the equation *true*, we say that the equation has one solution.

If there are certain values of the variable that make the equation *true*, we say the equation is true on a specific **domain (x-values; input).**

Exploratory Exercise #2:

Find the solution to the following equation: $2c + 2c + 2 = 4c + 2$

$$\begin{aligned} 4c + 2 &= 4c + 2 \\ -2 \quad & \quad -2 \\ 4c &= 4c \\ \div 4 \quad & \div 4 \\ c &= c \end{aligned}$$

Same expression on the left and right side of the equal sign.

There are infinite values for x that makes this equation true.

Sum It Up #2:

If there are many values of the variable that make the equation *true*, we say that the equation has many or infinite solutions! We call these equations identities.

OBJECTIVE: SWBAT determine when an algebraic fraction is undefined and its solution set.

Exploratory Exercise #3:

Find the solution to the following equation: $5(x+9)+3=5x+1$

$$5x+40+3=5x+1$$
$$5x+43=5x+1$$
$$-5x \qquad -5x$$
$$43=1$$ This is a false statement

There are **no** values for x that would make this equation true.

Sum It Up #3:

If there are no values of the variable that make the equation *true*, we say that the equation has no solutions!

Practice:

N = No Solution (None)

O = One Solution (One)

M = Many or Infinite Solutions (Many)

1) Solve and classify the following equations.
2) Circle the letter that is appropriate for each equation.
3) Check all solutions using substitution.

N O M $4x+7=7$	N O M $3(x-1)=2x+9$
N O M $2x+8=2(x+4)$	N O M $-2(x+1)=-2x+5$

OBJECTIVE: SWBAT determine when an algebraic fraction is undefined and its solution set.

Practice:

Given the following solutions, state whether their solutions are no solution, infinite solution, or one solution.

1) $x + 2 = x + 5$ ________________ **2)** $3x = 3x$ ________________

3) $x = -2$ ________________ **4)** $x + 8 = 2x + 8$ ________________

5) $x = 0$ ________________ **6)** $17 = -17$ ________________

Quick Assessment:

Solve and classify the following equation and circle its solution set.

N O M

$$3(x + 6) - 21 = 3x - 2$$

Literal Equations:

Literal equations contain more than one variable. Focus on the variable you need to solve for and you will be asked to solve "in terms" of the other variable.

Example: $A = \frac{1}{2}bh$

OBJECTIVE: SWBAT determine when an algebraic fraction is undefined and its solution set.

How to Rearrange Formulas:

Rearrange using the appropriate order to solve for ***x***:

Step 1:	Draw a line at the equal sign to separate the equation.	Solve for x: $y = mx + b$ $-b \quad -b$ $y - b = mx$ $\div m \quad \div m$ $\frac{y-b}{m} = x$
Step 2:	Identify the variable that you would need to solve for.	
Step 3:	Apply the inverse operation to **BOTH** sides of the equal sign, but keep the answer in terms of each letter. Inverse (Opposite) Operations: 1) Distribute/Combine 2) Add/Subtract 3) Multiply/Divide	
Step 4:	Repeat step 3, until you solve the equation.	

Practice:

The formula for the sum of the degree measures of the interior angles of a polygon is $S = 180(n - 2)$. Solve for n, the number of sides of the polygon in terms of S.	Solve the equation for x in terms of a. $4(ax + 3) - 3ax = 25 + 3a$

OBJECTIVE: SWBAT determine when an algebraic fraction is undefined and its solution set.

The formula for the area of a triangle is $A = \frac{1}{2}bh$. Express b in terms of h.	If the formula for the perimeter of a rectangle is $P = 2w + 2l$. Solve for length in terms of width.
The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. The radius, r, of the cone may be expressed as (1) $\sqrt{\frac{3V}{\pi h}}$ (3) $3\sqrt{\frac{V}{\pi h}}$ (2) $\sqrt{\frac{V}{3\pi h}}$ (4) $\frac{1}{3}\sqrt{\frac{V}{\pi h}}$	The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Express b_1 in terms of A, h, and b_2.

Quick Assessment:

Rearrange the following by solving for x.

$$ax + 3b = 2f$$

Standards: A.REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

OBJECTIVE: SWBAT identify and apply the operation to solve polynomial word problems.

Name: ______________________________ Date: __________________

Lesson 10: Linear Word Problems

Polynomial Word Problems:

After reading your word problem, identify your variable State clearly what you want each variable to represent. Written verbal expressions can be represented as more than one correct numeric expressions.

Ask Yourself:

1. What do I know?
2. What am I looking for?
3. Which operation do I use?

Example:

Then length of a window is represented by x^2, the width of the window is 6 less than the length. Express the area of the window as a polynomial in standard form.

$l = x^2$ $\quad$ $w = x^2 - 6$

$$A = l \cdot w$$
$$x^2(x^2 - 6)$$
$$x^4 - 6x^2$$

OBJECTIVE: SWBAT identify and apply the operation to solve polynomial word problems.

Practice:

If n represents a number, write an expression for 5 subtracted from the product of n and 17.	Jax traveled $-x^2 - 4x + 8$ miles in the morning and $2x^2 - 2x + 9$ miles in the afternoon. Write a polynomial expression to represent the total number of miles he traveled.
Sarah has a driveway whose length is represented by the expression $7x^2 - 4x + 9$ and whose width is represented by the expression $x + 3$. Find the perimeter of the driveway.	Fred is given a rectangular piece of paper. The length of Fred's piece of paper is represented by $2x^2 - x + 2$ and the width is represented by $3x^2$. Represent the area of the paper as a polynomial in standard form.

OBJECTIVE: SWBAT identify and apply the operation to solve polynomial word problems.

Creating Equations to Solve Word Problems:

1) Write 'let' statements for variables that define quantities.
2) Write an equation based on the desired variable.
3) Solve the equation.
4) Check the solution.

Example: Half of $\boldsymbol{k}$ minus 5 is the sum of $\boldsymbol{k}$ and 13. Find $\boldsymbol{k}$.

$\frac{k}{2}$ -5 $=$ $+$ k 13

$$\frac{k}{2} - 5 = k + 13$$

$$-\frac{1}{2}k - 5 = 13$$

$$-\frac{1}{2}k = 8$$

$$\times -2 \quad \times -2$$

$$k = -16$$

Practice:

Marvin paid an entrance fee of $\$5$ plus an additional $\$1.25$ per game at a local arcade. Altogether he spent $\$26.25$. Write and solve an equation to determine how many games Marvin played.

OBJECTIVE: SWBAT identify and apply the operation to solve polynomial word problems.

Consecutive Number Problems:

Definitions:

Consecutive Numbers: numbers that appear one after the other in a sequence (i.e. 16, 17, 18) and are represented by $\boldsymbol{n}, \boldsymbol{n+1}, \boldsymbol{n+2}$, etc. where $\boldsymbol{n}$ is any integer

Consecutive Odd/Even Numbers: numbers that are every other number in a sequence (i.e. 23, 25, 27) and are represented by $\boldsymbol{n}, \boldsymbol{n+2}, \boldsymbol{n+4}$, etc. where $\boldsymbol{n}$ is any integer

Example: The sum of four consecutive numbers is $\mathbf{-26}$. What are the numbers?

First Number: x
Second Number: $x+1$
Third Number: $x+2$
Fourth Number $x+3$

$$x+x+1+x+2+x+3=-26$$
$$4x+6=-26$$
$$-6 \quad -6$$
$$4x=-32$$
$$\div 4 \quad \div 4$$

$x=-8$ First Number
$x=-7$ Second Number
$x=-6$ Third Number
$x=-5$ Fourth Number

Perimeter Problems:

Warning: the word 'than' indicates a comparison between the two sides

Example: The width of a rectangle is $\mathbf{7}$ less than twice the length. If the perimeter of the rectangle is $\mathbf{43.6}$ inches, what is the length and width of the rectangle?

$length = l$
$width = 2l - 7$

$$l+l+2l-7+2l-7=43.6$$
$$6l-14=43.6$$
$$+14 \quad +14$$
$$6l=57.6$$
$$\div 6 \quad \div 6$$
$$l=9.6\ inches$$
$$w=2(9.6)-7=12.2\ inches$$

OBJECTIVE: SWBAT identify and apply the operation to solve polynomial word problems.

Practice:

Create an equation for the word problem and solve.

1) Kendal bought x boxes of cookies to bring to a party. Each box contains 12 cookies. She decides to keep two boxes for herself. She brings 60 cookies to the party. Write and equation to determine the number of boxes x Kendal bought. Then solve

2) A number increased by 5 and divided by the product of 2 and the number is equal to 75.

Quick Assessment:

At the beginning of her mathematics class, Mrs. Smith gives a warm-up problem. She says, "I am thinking of a number such that 6 less than the product of 7 and this number is 85." Determine which number Mrs. Smith is thinking of.

OBJECTIVE: SWBAT identify and apply the operation to solve polynomial word problems.

Describing Your Mathematical Processes:

Describe the property used to convert the equation from one line to the next:

$x(1-x)+2x-4=8x-24-x^2$	Given Equation
$x-x^2+2x-4=8x-24-x^2$	Distributive Property
$x+2x-4=8x-24$	Cancel Like Terms ($-x^2$)
$3x-4=8x-24$	Combine Like Terms
$3x+20=8x$	Add 24 to both sides
$20=5x$	Subtract $3x$ from both sides
$4=x$	Divide by 5 on both sides

Practice:

Solve the equation for x. For each step, describe the operation used to convert the equation.

$\mathbf{3x - [8 - 3(x - 1)] = x + 19}$

OBJECTIVE: SWBAT identify and apply the operation to solve polynomial word problems.

Practice:

Solve each equation for x. For each step, describe the operation and/or properties used to convert the equation.

1) $5(2x - 4) - 11 = 4 + 3x$

____

____

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____

2) $7x - [4x - 3(x - 1)] = x + 12$

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____

Standards: Standard: HSA-SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. HSA.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Create equations that describe numbers or relationships.

OBJECTIVE: SWBAT identify the inverse operations needed to solve an inequality.

Name: ______________________ Date: ______________

Lesson 11: Solving & Graphing Inequalities

Inequality Symbols:

$a < b$	a is less than b
$a \geq b$	a is greater than or equal to b
$a \leq b$	a is less than or equal to b
$a > b$	a is greater than b

How To Solve Inequalities:

Step 1: Step 2: Step 3:	Draw a line to separate the inequality. Use inverse operations to isolate the variable and follow this procedure: CD: Combine/Distribute AS: Addition/Subtraction MD: Multiplication/Division* Remember to BRING DOWN the inequality sign.	$2x + 3 \geq 19$

*EXCEPTION to the RULE:

When multiplying or dividing by a ***negative*** number, you must ***FLIP*** the inequality symbol.

A) $-\frac{x}{12} \leq \frac{1}{4}$

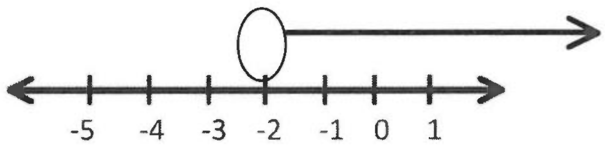
B) $-5x < 30$

OBJECTIVE: SWBAT identify the inverse operations needed to solve an inequality.

How to Graph the Solutions to Inequalities:

1) Label the numbers on the number line. (About 10 numbers)
   a. Whatever numbers they are close to…
2) Determine if it should be an open or closed circle.
   a. Open: $<$ or $>$
   b. Closed: $\leq$ or $\geq$
3) Draw the circle on the number.
4) Draw an arrow in the proper direction.
   a. To the Left: $<$ or $\leq$
   b. To the Right: $>$ or $\geq$

Example: If this was your solution how would you graph it? $x > -2$



Guided Practice:

Solve each inequality. Express each solution in set notation and graphically on a number line.

a) $5q + 10 > 20$

b) $6x - 30 \geq 30$

OBJECTIVE: SWBAT identify the inverse operations needed to solve an inequality.

Practice:

Solve and graph each inequality.

1) $2x + 4 \geq 24$

2) $7n - 1 > -169$

3) $2(n + 5) \leq -2$

4) $\frac{n}{3} + 8 \geq 9$

5) $x + 6 \leq 5$

6) $8n - 12 < 20$

OBJECTIVE: SWBAT identify the inverse operations needed to solve an inequality.

How To Solve Inequalities:

Step 1:	Draw a line to separate the inequality.	$8x - 2 > -9 + 7x$
Step 2:	Solve the inequality Follow the same rules: CD: Combine/Distribute AS: Addition/Subtraction MD: Multiplication/Division*	
Step 3:	Repeat step 2 until the variable is isolated (alone).	
Step 4:	Remember to BRING DOWN the inequality sign.	
***DON'T FORGET:**	***Flip the sign if you MULTIPLY or DIVIDE by a NEGATIVE!**	

Practice:

Solve each inequality. Express each solution graphically on a number line.

$-2(x + 4) > 6x - 2$	$-11 \geq 6 - 2n - 5$

OBJECTIVE: SWBAT identify the inverse operations needed to solve an inequality.

Solve the inequalities and plot the solution on a number line.

$-2 \geq 4p + 6 + 4$	$2(6 + 4n) \geq 12 - 8n$
$7(3x + 10) < 19x$	$3 \leq \frac{5x + 16}{12}$

OBJECTIVE: SWBAT identify the inverse operations needed to solve an inequality.

Closing Assessment:

Solve the inequality and plot the solution on the given number line.

$$6x + 2 + 6x < 14$$

Practice:

Solve each inequality and graph the solution.

$2(6x - 5) < 14$	$\frac{3x - 8}{7} \geq 1$

Standards: HSA.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

OBJECTIVE: SWBAT describe the solutions of a linear inequality.

Name: ______________________________ Date: ________________

Lesson 12: Compound Inequalities

Solution	Steps
Determine the largest integer value of a: $-2 - a - 7 > -12$	**1.** Solve the linear inequality. *REMEMBER TO CHANGE THE DIRECTION OF THE INEQUALITY IF WE MULTIPLY OR DIVIDE BY A NEGATIVE. **2.** Analyze your solutions: • $a <$ means all numbers LESS THAN a • $a >$ means all numbers GREATER THAN a • $a \leq$ means all numbers a and BELOW • $a \geq$ means all numbers a and ABOVE

Practice:

1) Determine the largest integer value of x: $6x + 2 + 6x < 14$

2) Given $-p - 4p > -10$, determine the largest integer value of p.

OBJECTIVE: SWBAT describe the solutions of a linear inequality.

3) Determine the smallest integer value of r: $5(6+3r)+7 \geq 127$

4) Given $a-6 \leq 15+8a$, determine the smallest integer value of a.

5) Determine the largest integer value of k: $28-k \geq 7(k-4)$

OBJECTIVE: SWBAT describe the solutions of a linear inequality.

Compound Inequalities:

A **compound inequality** is two simple inequalities joined by the word "and" or "or."

Solve and graph the following inequality. $3 < 2m - 1 < 9$	Step 1: "AND" separate the compound inequality into two inequalities. $3 < 2m - 1$ and $2m - 1 < 9$ Step 2: Solve both inequalities, treat the inequality symbol like an $=$. *Flip the inequality if you divide or multiply by a **negative**. Step 3: Graph the compound inequality on the number line, shade in the appropriate direction.

Using the word **"and"** (where both inequalities are true)

Solve and graph the solution set of: $2x - 3 \leq 9$ and $x + 2 \geq 4$.	Solve and graph the solution set of $2x - 1 < 5$ and $2x - 1 > -3$

OBJECTIVE: SWBAT describe the solutions of a linear inequality.

Using the word **"or"**(where one or both of the inequalities are true)

Solve and graph the solution set of:	Solve and graph the solution set of:
$5 + x < 7$ or $x - 3 > 5$	$3k + 7 < 7$ or $-4k + 5 < 1$

Practice:

Solve and graph each of the following compound inequalities.

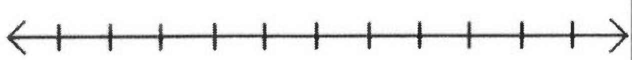
a) $m - 2 < -8$ or $\frac{m}{8} > 1$

b) $-1 + 5n > -26$ and $7n - 2 \leq 12$

OBJECTIVE: SWBAT describe the solutions of a linear inequality.

c) $5x - 5 > -7x - 5$ or $3x + 5 \leq x - 1$

d) $8x + 8 \geq -64$ and $-7 - 8x \geq -79$

1. $p + 4 \leq 1$ or $p - 1 \geq 1$ 	**2.** $9 - 12r \geq -99$ and $-2r - 4 < -12$ 

~~OBJECTIVE: SWBAT describe the solutions of a linear inequality.~~

Solutions to Linear Inequalities:

We may describe the solution to any inequality by graphing the solution on a number line or writing it in interval notation. An interval is a connected subset of numbers.

$<$ **Less Than** $<$	$>$ **Greater Than** $>$
Interval Notation: ( ) On a Number Line: ○	Interval Notation: ( ) On a Number Line: ○
$\leq$ **Less Than or Equal To** $\leq$	$\geq$ **Greater Than or Equal To** $\geq$
Interval Notation: [ ] On a Number Line: ●	Interval Notation: [ ] On a Number Line: ●

Example:

Given the inequality, $x > -3$:

x is the set of numbers greater than -3, which could be -2, -1, 0, 1, 2, 3, etc.

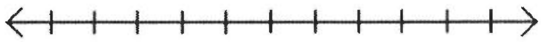
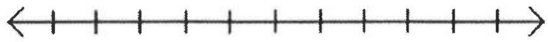
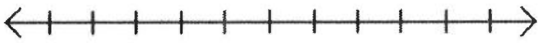
Interval Notation: (-3, ∞)

Practice:

For each of the following, graph the inequality on a number line and write the solution set in interval notation.

a) $-3 \leq x < 5$	b) $x \geq -2$
	
Interval Notation: ____________________	Interval Notation: ____________________

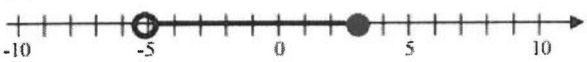
OBJECTIVE: SWBAT describe the solutions of a linear inequality.

c) $x < -5$ or $x \geq 0$  Interval Notation: ____________	d) $x < 9$  Interval Notation: ____________
e) $-6 < x \leq 4$  Interval Notation: ____________	f) $x \leq 1$ or $x > 3$  Interval Notation: ____________

Closing Assessment:

Write the graph's solution set in interval notation.

(a)



(b)

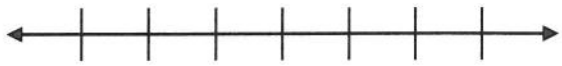
Interval Notation: ____________

Interval Notation: ____________

OBJECTIVE: SWBAT describe the solutions of a linear inequality.

Enrichment:

Ex 1)

Inequality	**Number Line Graph**	**Interval Notation**
x < 4 "x is less than 4"		
x > - 2 "x is greater than -2"		
x ≤ -1 "x is less than or equal to -1"		
x ≥ 0 "x is greater than or equal to 0"		

Ex 2) For the graph below, write:

A) Inequality: ______________________

B) Interval Notation: ________________

Ex 3) Which of the following represents the compound inequality $-5 \leq x < 19$?

(A) $(-5,19)$ (C) $[-5,19)$

(B) $(-5,19]$ (D) $[-5,-19]$

Standards: A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

OBJECTIVE: SWBAT create inequalities by identifying the inequality symbol that corresponds with a word or phrase.

Name: ______________________________ Date: ________________

Lesson 13: Linear Inequality Word Problems

Translating Inequalities:

$<$	$>$
Is less than Is smaller than Is fewer than Below	Is greater than Is larger than Is more than Above
$\leq$	$\geq$
Maximum At most Is not greater than Is not more than	Minimum At least Is not less than Is not smaller than

Translating Inequalities:

1. Identify the unknown (what you are looking for) and assign a variable to it (for example: "each" and "a number").
2. Look for key words to indicate where the inequality sign should be placed. (Hint: use the activity on the last page)
3. Translate by annotating.
4. Put it all together! Write the sentence as an inequality.

Example:

12 increased by a number is at least 50.

$12 \quad + \quad x \quad \geq \quad 50$

OBJECTIVE: SWBAT create inequalities by identifying the inequality symbol that corresponds with a word or phrase.

Practice:

Translate the following sentences into an inequality. Then solve, graph, and write the solution set in interval notation.

a) Five is greater than 6 decreased by twice a number.

Interval Notation:

b) The sum of three times a number and twelve is fewer than twenty.

Interval Notation:

c) A number plus 7 is not less than 35.

Interval Notation:

d) Carlo's allowance is $2 per chore but he can make no more than $16.

Interval Notation:

OBJECTIVE: SWBAT create inequalities by identifying the inequality symbol that corresponds with a word or phrase.

e) Ten more than a number is no more than 62.

Interval Notation:

____

f) The sum of four and twice a number is at least 52.

Interval Notation:

____

g) The sum of five and twice a number is at most 17.

Interval Notation:

____

h) If 5 times a number is decreased by 12, the difference is above 9.

Interval Notation:

____

OBJECTIVE: SWBAT create inequalities by identifying the inequality symbol that corresponds with a word or phrase.

Guiding Questions:

When translating inequalities, ask yourself the following:

1. What do I know?
2. What am I looking for?
3. Which inequality symbol do I use?

Creating Inequalities to Solve Word Problems:

1) Read the word problem.
2) Write 'let' statements for variables that define quantities.
3) Write an inequality based on the desired variable.
4) Solve the inequality.

Practice:

a) Jason's part-time job pays him \$155 a week. If he has already saved \$375, what is the minimum number of weeks he needs to work in order to have enough money to buy a dirt bike for \$900?

b) Connor wants to attend the town carnival. The price of admission to the carnival is \$4.50, and each ride costs an additional 79 cents. Connor can spend at most \$16.00 at the carnival. Determine the maximum number of rides, *r*, Connor can go on.

OBJECTIVE: SWBAT create inequalities by identifying the inequality symbol that corresponds with a word or phrase.

c) Sally rented a car for $45.00 a week plus $0.12 for each mile the car is driven. What is the greatest number of miles Sally can drive the car if she wishes to spend at most $105 in one week?

d) Paul earns $25.00 per hour at his job. Each day he spends $10.00 for lunch. Today he wants to take home at least $215.00 after paying for his lunch. How many hours will he have to work to achieve his goal?

e) An online music club has a one-time registration fee of $13.95 and charges $0.49 to buy each song. If Emma has $50.00 to join the club and buy songs, what is the maximum number of songs she can buy?

OBJECTIVE: SWBAT create inequalities by identifying the inequality symbol that corresponds with a word or phrase.

Closing Assessment:

Translate the following inequalities. Then, solve and graph the outcomes.

a) Mrs. Smith wrote "Six less than three times a number is greater than fifteen" on the board. If x represents the number, write an inequality and solve.

b) Chelsea has \$45 to spend at the fair. She spends \$20 on admission and \$15 on snacks. She wants to play a game that costs \$0.65 per game. Write an inequality to find the maximum number of times, g, Chelsea can play the game. Using this inequality, determine the maximum number of times she can play the game.

Standards: CCSS.Math.Content.HSA-REI.A.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. CCSS.6.EE.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real- world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Standards: CCSS.6.EE.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real- world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

OBJECTIVE: SWBAT create and identify functions.

Name: ______________________________ Date: ________________

Lesson 14: Types of Functions

Functions:

A **function** is a clearly defined **rule** that converts an **input** into **exactly one output**. These rules often come in the form of: (1) equations, (2) graphs, (3) tables, and (4) verbal descriptions.

Example:

***Exercise* #1:** Consider the function rule: multiply the input by two and then subtract one to get the output.

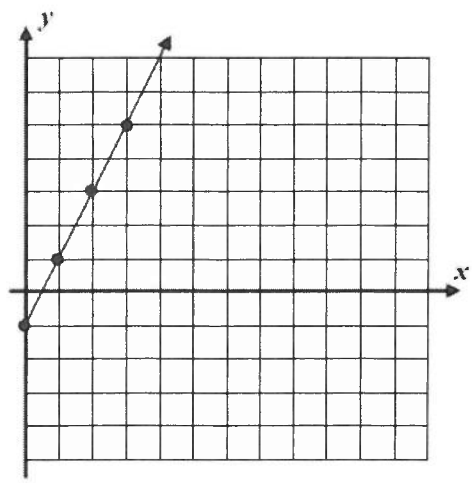
(a) Fill in the table below for inputs and outputs. Inputs are often designated by x and outputs by y.

Input x	Calculation	Output y
0	$=2(0)-1=0-1=-1$	-1
1	$=2(1)-1=2-1=1$	1
2	$=2(2)-1=4-1=3$	3
3	$=2(3)-1=6-1=5$	5

(b) Write an equation that gives this rule in symbolic form.

$y=2x-1$

(c) Graph the function rule on the graph paper shown below. Use your table in (a) to help.



***Exercise* #2:** In the function rule from #1, what input would be needed to produce an output of 17? Why is it harder to find an input when you have an output than finding an output when you have an input?

$2x-1=17 \Rightarrow 2x-1+1=17+1$

$2x=18 \Rightarrow \frac{2x}{2}=\frac{18}{2}$

$x=9$

Functions rules tell you how to convert an input into an output. If we are given the output, though, the rule needs to be reversed, which can be challenging, depending on how complex the rule is.

OBJECTIVE: SWBAT create and identify functions.

Practice:

Function rules do not always have to be numerical in nature, they simply have to return a single output for a given input.

The table below gives a rule that takes as an input a neighborhood child and gives as an output the month he or she was born in.

(a) Why can we consider this rule a function?

Child	Birth Month
Max	January
Evin	April
Zeke	May
Rosie	February
Niko	May

(b) What is the output when the input is Rosie?

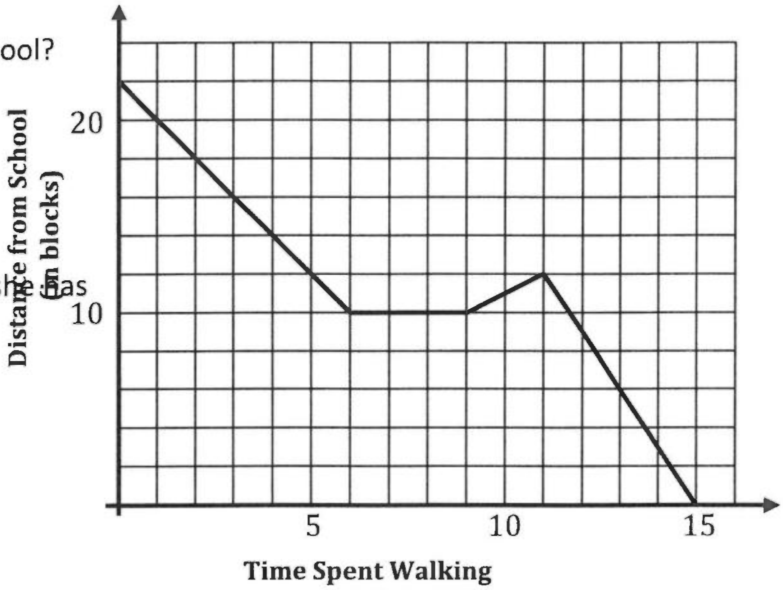
(c) Find all inputs that give an output of May. Why does this *not* violate the definition of a function even though there are two answers?

OBJECTIVE: SWBAT create and identify functions.

Practice:

Functions are useful because they can often be used to **model** things that are happening in the real world.

Charlene heads out to school by foot on a fine spring day. Her distance from school, in blocks, is given as a function of the time, in minutes, she has been walking. This function is represented by the graph given below.



(a) How far does Charlene start off from school?

(b) What is her distance from school after she has been walking for 4 minutes?

(c) After walking for six minutes, Charlene stops to look for her subway pass. How long does she stop for?

(d) Charlene then walks to a subway station before heading to school on the subway (a local). How many blocks did she walk to the subway?

(e) How long did it take for her to get to sc once she got on the train?

OBJECTIVE: SWBAT create and identify functions.

Vocabulary:

Coordinate Plane: the plane formed by two lines called the x-axis and y-axis.

It is divided into four quadrants.

x-axis: the horizontal number line; goes from left to right

y-axis: the vertical number line; goes down to up

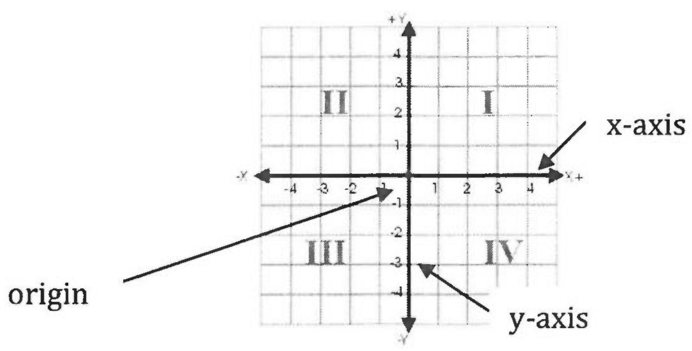
Ordered Pair: pair of numbers that represent a unique location on the coordinate plane.

The first value is the x coordinate.

The second value is the y coordinate.

Example: (2,3) the x coordinate is 2 and the y coordinate is 3.

Origin: the center of the coordinate plane. It has coordinates $(0, 0)$. It is the point where we always begin when plotting.

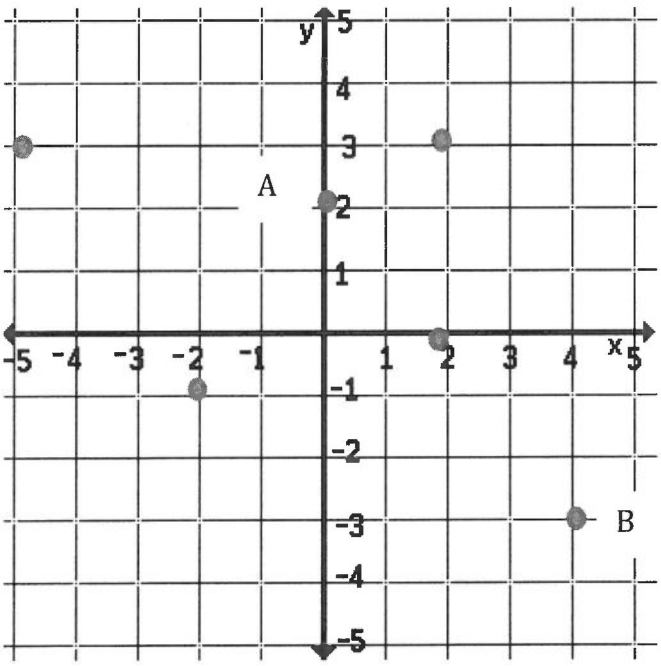


OBJECTIVE: SWBAT create and identify functions.

How to Plot Coordinate Points:

On the coordinate plane below, the following points are graphed. Continue labeling the points.

$A(0,2), B(4,-3), C(-2,-1), D(2,3), E(2,0)$, F (-5, 3)



Steps

1. Start at the origin

2. Look at the x value.
   If x is positive, move that many spaces right.
   If x is negative, move that many spaces left.

3. From that location, look at the y value.
   If y is positive, move that many spaces up
   If y is negative, move that many spaces down.

4. From this location, plot a point

Practice:

On the coordinate plane below, graph the following coordinate pairs and write the quadrant the point is in.

A (5, 2)

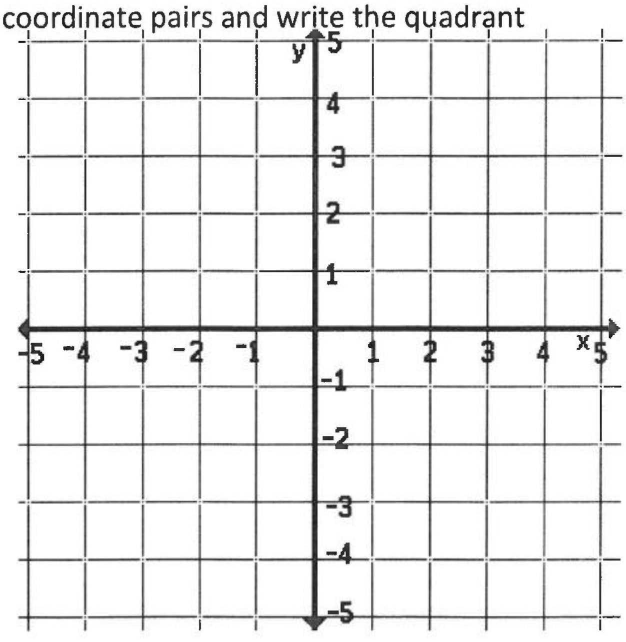
B (0, −3)

C (−2, 1)

D (−4, −5)

E (1, −1)

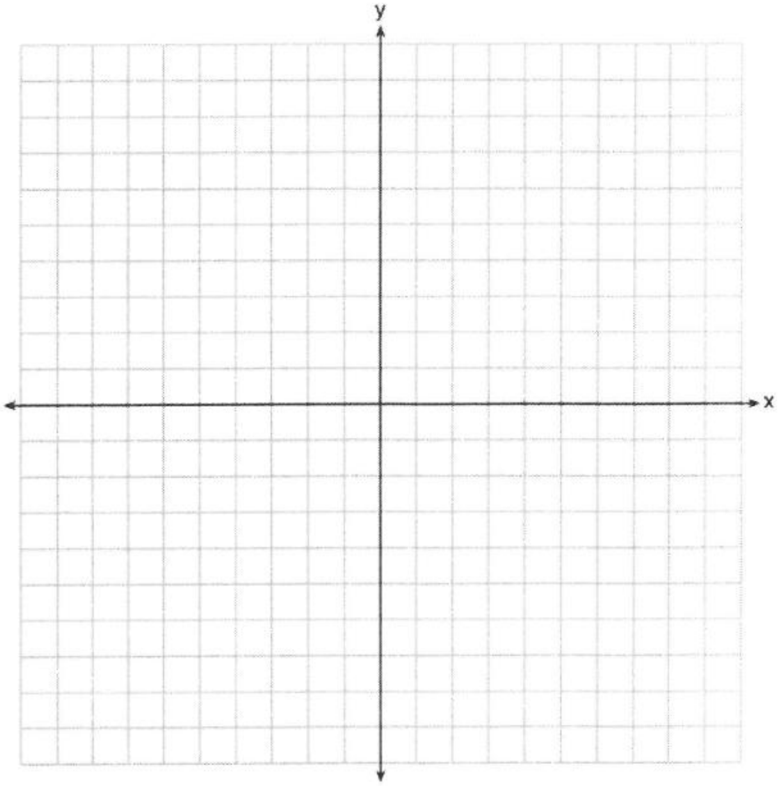
F (3, 0)



OBJECTIVE: SWBAT create and identify functions.

Practice:

On the coordinate plane, graph the following coordinate pairs and identify the quadrant each point is in.



A (2, 3)

B (4, -6)

C (-3, -5)

D (-4, 5)

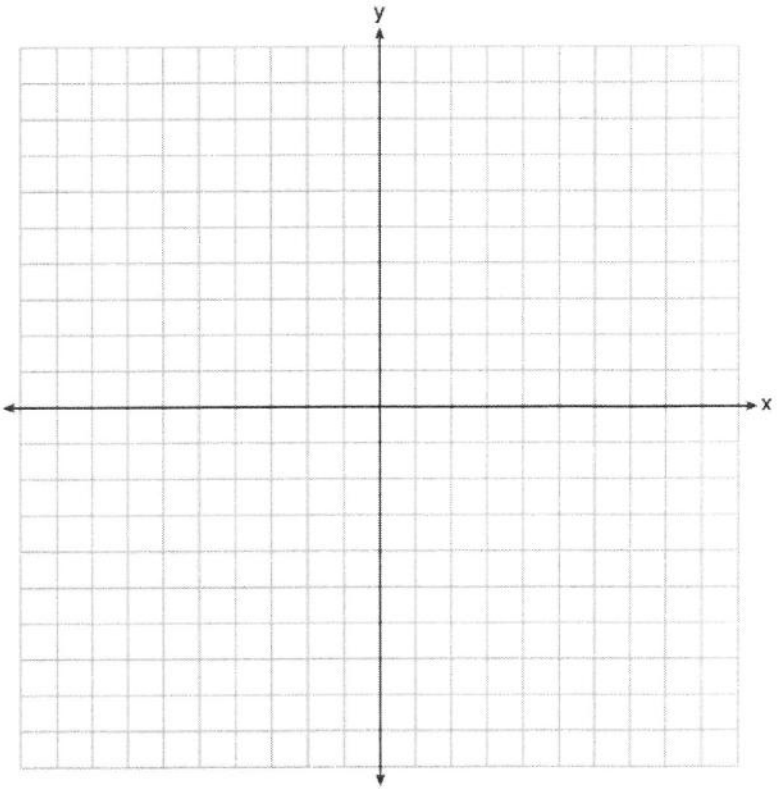
E (4, 6)

F (8, -2)

G (-5, -3)

H (-6, 4)

On the coordinate plane below, graph the following coordinate pairs and identify the quadrant each point is in.



I (5, 8)

J (4, -2)

K (-3, -3)

L (-4, 7)

M (8, 5)

N (8, -2)

O (-4, -4)

P (-6, 8)

OBJECTIVE: SWBAT create and identify functions.

All of our first elements of ordered pairs (**x-coordinates**) are called the **domain**.

All of our second elements of ordered pairs (**y-coordinates**) are called the **range**.

{ } are used for interval notation.

Example: $\{(0,1), (2,5), (-4,8), (-1,3)\}$ is a **set of ordered pairs.**

Example: If we have a **domain** of $\{-4, -1, 0, 2\}$, this is a **set of all x-values.**

Example: If we have a **range** as $\{1, 3, 5, 8\}$, this is a **set of all y-values.**

Relevant Vocabulary:

Coordinate Pairs: (x, y)

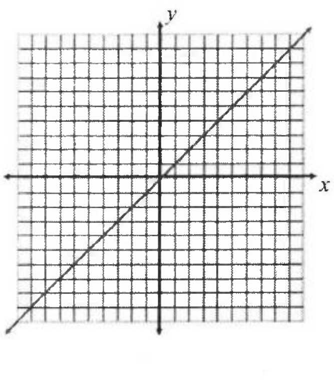
Relation: A relation is set of coordinate pairs.

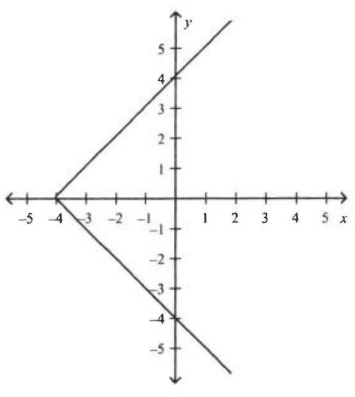
Function: A function is a set of coordinate pairs where each x-value is matched with only one y-value.

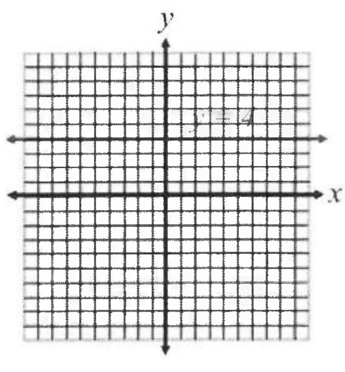
Vertical Line Test:

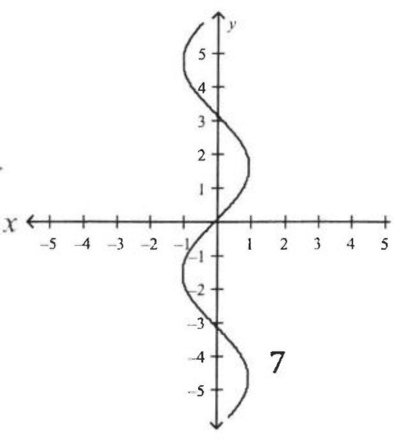
To determine if a relation is a function we can use the vertical line test. If any vertical line touches the given graph more than once, then it is NOT a function.

Which of the following are functions?









OBJECTIVE: SWBAT create and identify functions.

Modeled Practice: Determining Functions

Identify the domain and range. Determine whether or not the relation is a function.

Remember: a relation is a function **if** no x-values repeat; x values are different.

Given a set:

$g = \{(1,5),(2,6),(1,8),(2,9),(3,7)\}$

Domain: {1, 2, 1, 2, 3}

Range: {5, 6, 7, 8, 9}

Function? Yes No

Given a table:

Input	0	1	2	3	4	5
Output	1	2	4	8	16	32

Domain: {0, 1, 2, 3, 4, 5}

Range: {1, 2, 4, 8, 16, 32}

Function? Yes No

Practice:

Identify the domain and range. Determine whether or not the relation is a function.

Given a set:

$f = \{(1,7),(2,5),(3,6),(4,7)\}$

Domain:

Range:

Function? Yes No

Given a table:

x	-6	-4	-2	-4	12	35
$f(x)$	-6	-20	-42	8	16	-20

Domain:

Range:

Function? Yes No

OBJECTIVE: SWBAT create and identify functions.

A **function** is a set of ordered pairs in which x **cannot repeat.**

Example: In the set $\{(1,4),(3,6),(3,7)\}$ the x value does repeat. This is **not a function.**

Example: In the set $\{(0,4),(2,8),(3,4)\}$ does **not have repeating x values.** This is a function.

1) $\{(1,3),(-2,7),(3,-3),(4,5),(1,-3)\}$

Function: Yes / No

Why? ____________________

2) $\{(-9,3),(6,14),(0,0),(-1,-1),(1,2)\}$

Function: Yes / No

Why? ____________________

3) $\{(-3,-2),(-2,-1),(-1,0),(0,1),(1,2)\}$

Function: Yes / No
Why? ____________________

4) $\{(21,3),(7,0),(-14,-3),(-14,-40)\}$

Function: Yes / No
Why? ____________________

5)

x	y
−2	3
1	4
5	6
2	−1

Function: Yes / No

Why? ____________________

6)

x	y
2	3
4	2
2	−5
−6	−3

Function: Yes / No

Why? ____________________

7)

x	y
1	5
3	−6
1	−5
−2	−9

Function: Yes / No
Why? ____________________

8)

x	y
−1	2
3	6
6	2
−9	4

Function: Yes / No
Why? ____________________

OBJECTIVE: SWBAT create and identify functions.

Practice:

A) Which table represents a function?

(1)

x	2	4	2	4
f(x)	3	5	7	9

(3)

x	3	5	7	9
f(x)	2	4	2	4

(2)

x	0	−1	0	1
f(x)	0	1	−1	0

(4)

x	0	1	−1	0
f(x)	0	−1	0	1

B) Which of the following sets does not represent a function?

(1) $f = \{(-7,1),(-3,\ 0),(0,-2),(4,-2)\}$

(2) $g = \{(0,3),(2,3),(4,3),(6,3)\}$

(3) $h = \{(0,2),(2,4),(0,5),(4,5)\}$

(4) $j = \{(-12,\ -5),(-6,\ 7),(-2,4),(3,\ 9)\}$

Closing Assessment:

Given the relation:
$f = \{(-2,3),(0,0),(5,4),(7,3),(7,4)\}$

(a) State the domain: ______________________________

(b) State the range: ______________________________

(c) Is the relation a function? ______________________

(d) Why or why not? ______________________________

OBJECTIVE: SWBAT create and identify functions.

Function Notation

Equations can be written in a form called function notation. We use this as a quick way to evaluate functions for a given input (x value).

Example: **y = 2x – 8** → **f(x) = 2x – 8**

This is read as f of x equals 2 x minus 8

For every function, each x value is used as an input, to generate an output y value.

Example: Evaluate the function $f(x) = 3x + 1$ for $f(2)$.

Notice that our x was replaced with a 2. Therefore, we will substitute a 2 for each x in the function.

$f(x) = 3x + 1$ ← **The function**

$f(2) = 3(2) + 1$ ← **The input value is 2, so 2 was substituted in for x**

$f(2) = 6 + 1$ ← **Use Order of Operations to simplify and find the output**

$f(2) = 7$ ← **You must write your answer using function notation**

Our input (x) value of 2, generated an output (y) value of 7

We can write this as an **ordered pair** (x, y) as (2, 7)

Example: $f(x) = x + 7$ for $x = 5$
f(5) = 5 + 7
f(5) = 12
(5, 12)

OBJECTIVE: SWBAT create and identify functions.

Practice:

1) Given the function $g(x) = 3x - 8$ find:

$g(1)$	$g(-3)$	$g(0)$

2) Given the function $f(x) = x^2 - x$, find:

$f(-4)$	$f(-1)$	$f(7)$

3) Given the function $h(b) = 3b^2 - 2b - 1$, find

$h(-3)$	$h(0)$	$h(2)$

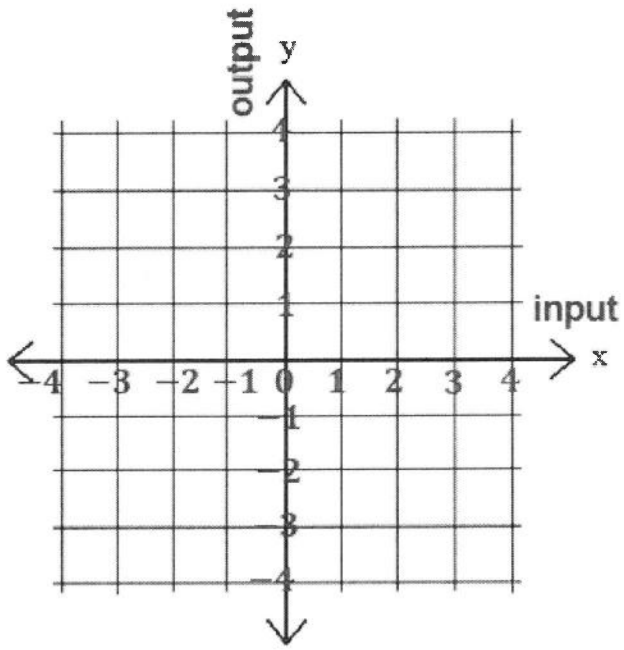
OBJECTIVE: SWBAT create and identify functions.

From a Graph:

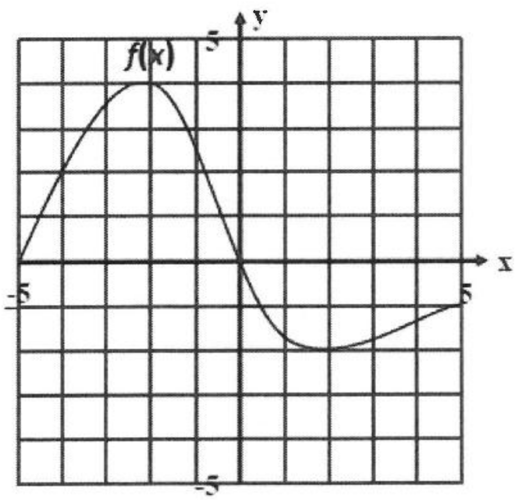
When finding the output value of a function from a graph, we use the x and y axes as a guide.

Our x-axis (←→) goes from left to right. To the left is negative to the right is positive.

Our y-axis goes from down to up. Down is negative and up is positive.



Directions: Given the graph of the function $f(x)$, find each of the following.



$f(-4) =$

$f(0) =$

$f(-5) =$

$f(2) =$

$f(5) =$

$f(-2) =$

OBJECTIVE: SWBAT create and identify functions.

Practice:

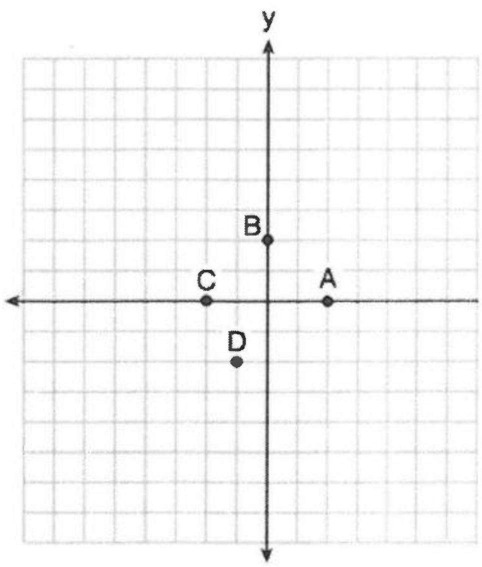
1) Boiling water at 212 degrees Fahrenheit is left in a room that is at 65 degrees Fahrenheit and begins to cool. Temperature readings are taken each hour and are given in the table below. In this scenario, the temperature, T, is a function of the number of hours, h.

h (hours)	0	1	2	3	4	5	6	7	8
$T(h)$ ($^\circ F$)	212	141	104	85	76	70	68	66	65

(a) Evaluate $T(2)$ and $T(6)$

(b) For what value of h is $T(h) = 76$
(Find the input value for the output value = 76)

2) The graph of $y = f(x)$ is shown below.



a) What point could be used to find $f(2)$?

2 is an ______ value

When $x = 2$, $y =$______

As a coordinate we write this as ________

$(2,0)$ is the letter ________.

b) What letter could be used to find $f(0)$?

What are the coordinates of this letter?

c) What letter could be used to find $f(-1)$?

What are the coordinates of this letter?

Standards: CC.F.IF.2: Functional Notation: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

Name: ______________________________ Date: ________________

Lesson 15: Average Rate of Change

Speed:

Average Rate of Change (Speed):

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example:

An airplane leaves New York City and heads toward Los Angeles. As it climbs, the plane gradually increases its speed until it reaches cruising altitude, at which time it maintains a constant speed for several hours as long as it stays at cruising altitude.

After flying for 32 minutes, the plane reaches cruising altitude and has flown 192 miles. After flying for a total of 92 minutes, the plane has flown a total of 762 miles. Determine the speed of the plane, at cruising altitude, in miles per minute.

OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

Practice:

Loretta and her family are going on vacation. Their destination is 610 miles from their home. Loretta is going to share some of the driving with her dad. Her average speed while driving is 55 mph and her dad's average speed while driving is 65 mph. The plan is for Loretta to drive for the first 4 hours of the trip and her dad to drive for the remainder of the trip. Determine the number of hour it will take her family to reach their destination.

After Loretta has been driving for 2 hours, she gets tired and asks her dad to take over. Determine, to the *nearest tenth of an hour,* how much time the family will save by having Loretta's dad drive for the remainder of the trip.

Practice:

1) A cell phone can receive 120 messages per minute. At this rate, how many messages can the phone receive in 150 seconds?

1) 48
2) 75
3) 300
4) 18,000

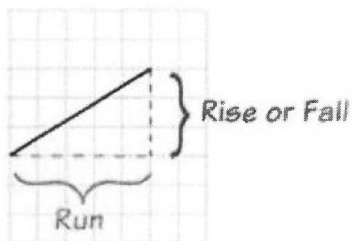
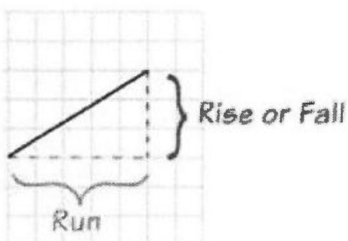
OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

2) It takes a snail 500 hours to travel 15 miles. At this rate, how many ours will it take the snail to travel 6 miles?

1) 0.18
2) 5.56
3) 150
4) 200

3) Joseph typed a 1,200-word essay in 25 minutes. At this rate, determine how many words he can type in 45 minutes.

4) In a game of ice hockey, the hockey puck took 0.8 second to travel 89 feel to the goal line. Determine the average speed of the puck in feet per second.

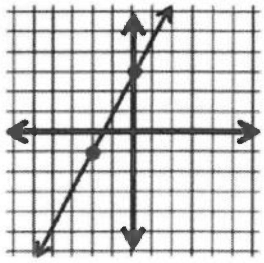
Vocabulary	**Definition**	**Example**
Rise	The vertical distance between two points on a line.	
Run	The horizontal distance between two points on a line.	

OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

Types of Slope:

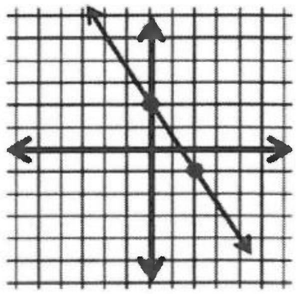
Positive Slope: As you read from left to right, the line increases.

POSITIVE



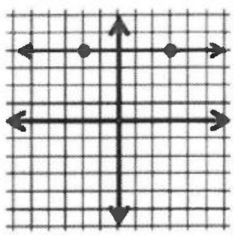
Negative Slope: As you read from left to right, the line decreases.

NEGATIVE



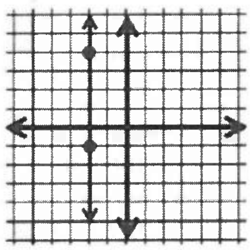
Zero Slope: As you read from left to right, the graph remains constant.

Zero



Undefined/No Slope: You cannot read the graph from left to right; it is vertical.

Undefined

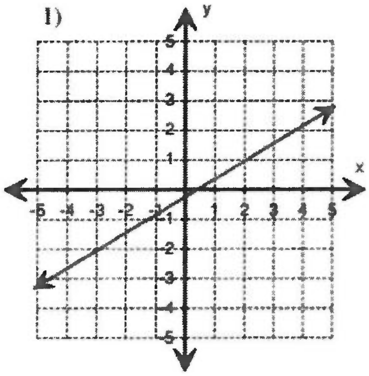


OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

Type of Slope:

Identify the type of slope in the given graphs (positive, negative, zero, or undefined).

1)



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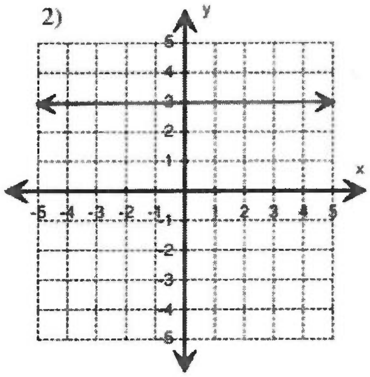
2)



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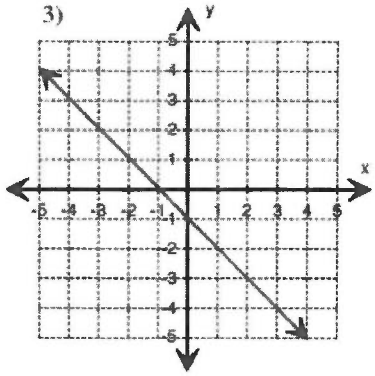
3)



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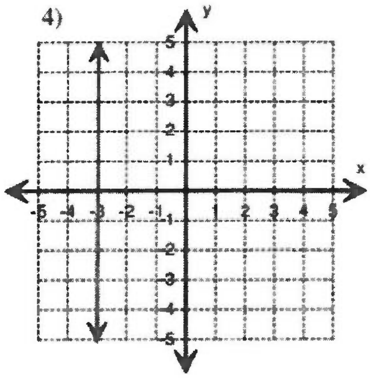
4)



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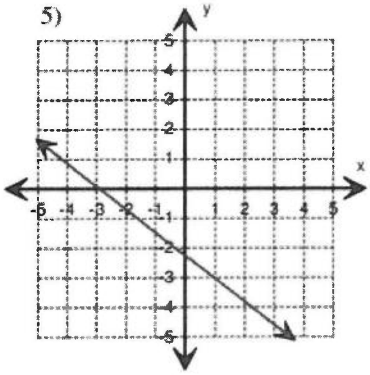
5)



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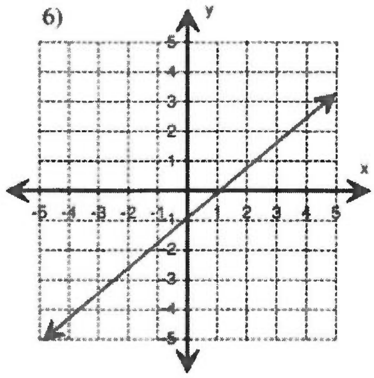

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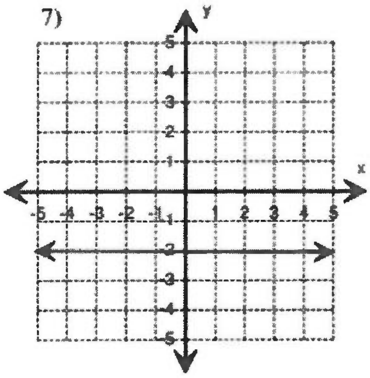

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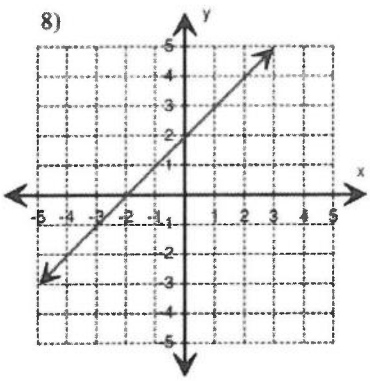
8)



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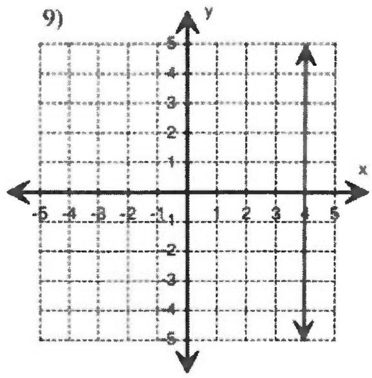
9)



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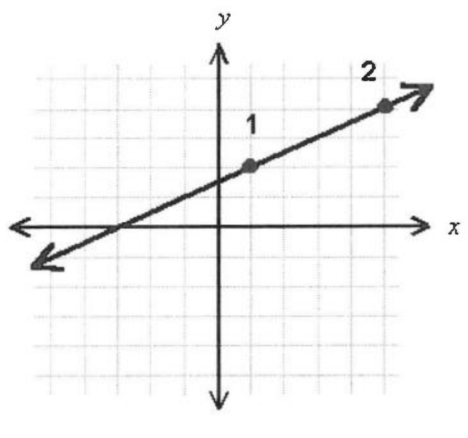
OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

How to Find Slope Using Rise Over Run:

Step 1:	Starting at the left of the line, trace the line until you come to a point on a corner. Label.	
Step 2:	Continue to trace until you come to a second point on a corner. Label.	
Step 3:	Count UP or DOWN: How many boxes until you reach a second point on a corner (rise). UP = $(+)$ DOWN = $(-)$	*Determine if* $(+)$*or* $(-)$
Step 4:	Count OVER to the RIGHT: How many boxes until you reach the same second point (run). RIGHT = $(+)$	$\frac{+ \text{ or } -}{(+)}$

Example:

Calculate the slope using: $\frac{rise}{run}$



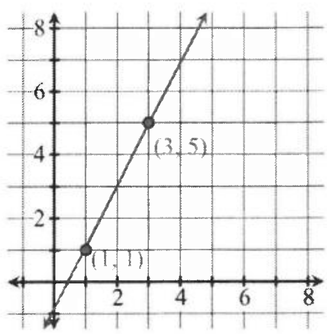
Rise = 2

Run = 4

Slope (m) = 2/4 = ½

Practice:

Find the slope of a line through that passes through the following points:



Rise = ________

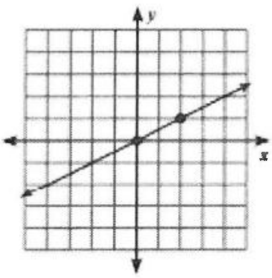
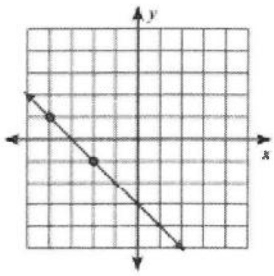
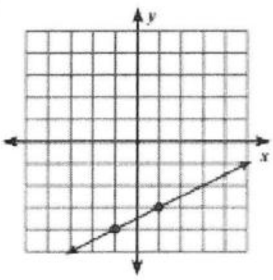
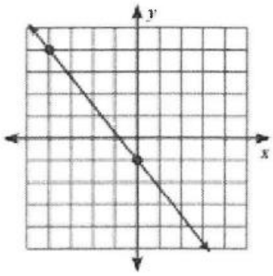
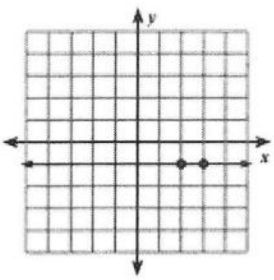
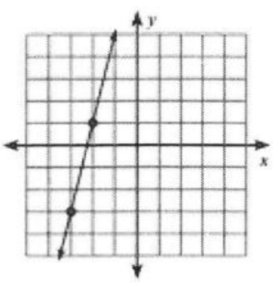
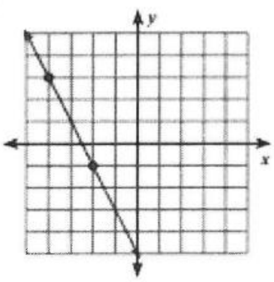
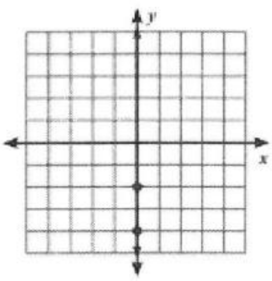
Run = ________

Slope (m) = ________

OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

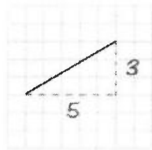
Practice:

Find the slope of the following lines using rise over run.

1) 	2) 
3) 	4) 
5) 	6) 
7) 	8) 

OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

What is Slope?

Vocabulary	Definition	Example
Slope Symbol: m	How steep a straight line is; the incline; the average rate of change	

Slope Formula:

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

How to Use the Slope Formula:

Example: Find the slope of a line through the coordinate pairs (-7, 17) and (-15, 16).

Step 1:	Identify and LABEL the x's and y's directly above the given numbers.	(-7, 17) and (-15, 16)
Step 2:	Substitute the numbers into the formula. $\frac{y_2 - y_1}{x_2 - x_1}$	$\frac{16 - 17}{-15 - (-7)}$
Step 3:	Solve! • Subtract the numerator. • Subtract the denominator. • Divide and simplify.	$\frac{-1}{-8} = \frac{1}{8}$

OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

Practice:

Solution	Steps
The table shows the average temperature (°F) for five months in a certain city. Find the rate of change from the 3rd month to the 7th month. Month: 2, 3, 5, 7, 8 Temp. (°F): 56, 56, 63, 71, 72	1. Write your rate of change formula. 2. Identify your coordinate points. 3. Substitute your coordinates into the formula $\frac{y_2 - y_1}{x_2 - x_1}$ 4. Write your answer in simplest form.

Month	2	3	5	7	8
Temp. (°F)	56	56	63	71	72

Practice:

1) The table shows the balance of a bank account on different days of the month.

Day	1	6	16	22	30
Balance ($)	550	285	210	210	175

a) What was the average rate of change from day 1 to day 16?

b) What was the average rate of change from day 6 to day 30?

OBJECTIVE: SWBAT determine the rate or speed of a situation from a word problem.

2) The table shows the average diameter of a pupil in a person's eye as he or she grows older. What is the average rate of change, in millimeters per year, of a person's pupil diameter from age 20 to age 80?

Age (years)	**Average Pupil Diameter** (mm)
20	4.7
30	4.3
40	3.9
50	3.5
60	3.1
70	2.7
80	2.3

3) Joey enlarged a 3-inch by 5-inch photograph on a copy machine. He enlarged it four times. The table below shows the area of the photograph after each enlargement.

Enlargement	0	1	2	3	4
Area (square inches)	15	18.8	23.4	29.3	36.6

What is the average rate of change of the area from the original photograph to the fourth enlargement, to the *nearest tenth*?

Closing Assessment:

Jason was asked to find the average rate of change between the two points (2,5) and (-1,3). His steps are shown below.

$\frac{2-(-1)}{5-3}$ [step 1]

$\frac{2+1}{2}$ [step 2]

$\frac{3}{2}$ [step 3]

Do you agree or disagree with Jason's answer? Explain your answer.

OBJECTIVE: SWBAT convert units of measure.

Name: ______________________________ Date: ________________

Lesson 16: Unit Conversions

Converting Units:

1. Set up a proportion with the given information.
2. Solve the proportion for the unknown.

Alternative: Multiply by a ratio.

If the units are going from Left → Right, multiply.
If the units are going from Right ← Left, divide.

REGENTS REFERENCE SHEET:

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Modeled Examples:

a) John has traveled a total of 4.5 miles. If there are 5,280 feet in each mile, how many feet did John travel?

4.5 miles(5280 ft/mile) = 23,760 feet *The unit of miles cancels.

b) If there are exactly 2.54 centimeters in each inch, how many centimeters are in one foot? Show the work that leads to your answer.

You need to know that there are 12 inches in one foot (it is not on the Regents reference sheet).

2.54 cm/inch (12 in/ft) = 30.28 cm/ft * The unit of inches cancels.

OBJECTIVE: SWBAT convert units of measure.

Practice:

1) A typical marathon is 26.2 miles. Allan averages 12 kilometers per hour when running in marathons. Determine how long it would take Allan to complete a marathon, to the nearest tenth of an hour. Justify your answer.

2) Faith wants to use the formula $C(f) = \frac{5}{9}(f - 32)$ to convert degrees Fahrenheit, f, to degrees Celsius, $C(f)$. If Faith calculated $C(68)$, what would her result be?

1) 20° Celsius

2) 20° Fahrenheit

3) 154° Celsius

4) 154° Fahrenheit

2) A local factory has to add a liquid ingredient to make their product at a rate of 13 quarts every 5 minutes. How many gallons per hour of the ingredient do they need to add?

4) How many centimeters are there in 1 yard if there are 2.54 centimeters per inch?

OBJECTIVE: SWBAT convert units of measure.

5) A high school track athlete sprints 100 yards in 15 seconds. Determine the number of feet per second the runner is traveling at.

6) Dan took 12.5 seconds to run the 100-meter dash. He calculated the time to be approximately

1. 0.2083 minute
2. 750 minutes
3. 0.2083 hour
4. 0.52083 hour

7) Peyton is a sprinter who can run the 40-yard dash in 4.5 seconds. He converts his speed into miles per hour, as shown below.

$$\frac{40\text{ yd}}{4.5\text{ sec}} \cdot \frac{3\text{ ft}}{1\text{ yd}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}}$$

Which ratio is *incorrectly* written to convert his speed?

1) $\frac{3\text{ ft}}{1\text{ yd}}$

2) $\frac{5280\text{ ft}}{1\text{ mi}}$

3) $\frac{60\text{ sec}}{1\text{ min}}$

4) $\frac{60\text{ min}}{1\text{ hr}}$

OBJECTIVE: SWBAT convert units of measure.

8) A construction worker needs to move 120 ft^3 of dirt by using a wheelbarrow. One wheelbarrow load holds 8 ft^3 of dirt and each load takes him 10 minutes to complete. One correct way to figure out the number of hours he would need to complete this job is

1) $\frac{120\text{ft}^3}{1} \cdot \frac{10\text{ min}}{1\text{ load}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ load}}{8\text{ ft}^3}$

2) $\frac{120\text{ft}^3}{1} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{8\text{ ft}^3}{10\text{ min}} \cdot \frac{1}{1\text{ load}}$

3) $\frac{120\text{ft}^3}{1} \cdot \frac{1\text{ load}}{10\text{ min}} \cdot \frac{8\text{ ft}^3}{1\text{ load}} \cdot \frac{1\text{ hr}}{60\text{ min}}$

4) $\frac{120\text{ft}^3}{1} \cdot \frac{1\text{ load}}{8\text{ ft}^3} \cdot \frac{10\text{ min}}{1\text{ load}} \cdot \frac{1\text{ hr}}{60\text{ min}}$

Unit Rates:

Unit Rates: rate over 1; simplified slope to value over 1

Writing Rates:

- The speed limit is 50 miles per hour.
- You can seat 4 people per car.
- A bottle of soda costs $1.49 per gallon.

Practice:

a) A two-inch grasshopper can jump a horizontal distance of 40 inches. An athlete, who is five feet nine, wants to cover a distance of one mile by jumping. If this person could jump at the same ratio of body-length to jump-length as the grasshopper, determine, to the *nearest jump*, how many jumps it would take this athlete to jump one mile.

OBJECTIVE: SWBAT convert units of measure.

b) The distance traveled is equal to the rate of speed multiplied by the time traveled. If the distance is measured in feet and the time is measured in minutes, then the rate of speed is expressed in which units? Explain how you arrived at your answer.

Closing Assessment:

If there are 1000 grams in a kilogram and 454 grams in a pound, how many pounds are there per kilogram? Round to the nearest tenth of a pound.

Standards: CCSS.Math.Content.N-Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

OBJECTIVE: SWBAT create the graph of a linear function.

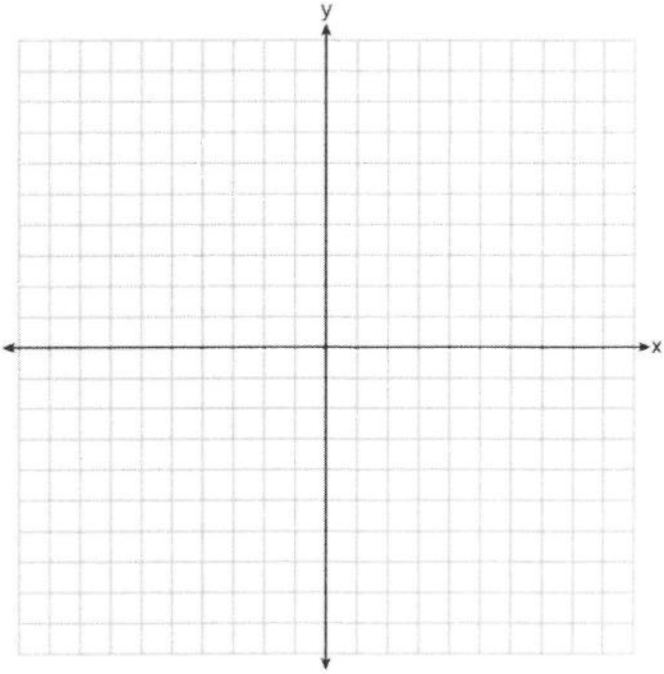
Name: ______________________________ Date: __________________

Lesson 17: Graphing Linear Functions

Do Now:

a) The table for the linear function $y = -2x + 4$ is shown in the table below. Graph these points on the set of axes provided.

x	y
−3	10
−2	B
−1	6
0	4
1	2
2	0
3	−2



b) Connect your points using a straight line.

c) What is the slope of the line you drew?

d) Circle the point where the line touches the y-axis.

e) Relate your answers to (c) part (d) to the linear equation we started with $y = -2x + 4$

OBJECTIVE: SWBAT create the graph of a linear function.

Linear Functions:

A linear function has a degree of 1. Equations of lines often appear in slope-intercept form. Remember: intercept means to cross.

$$y = mx + b$$

m: slope

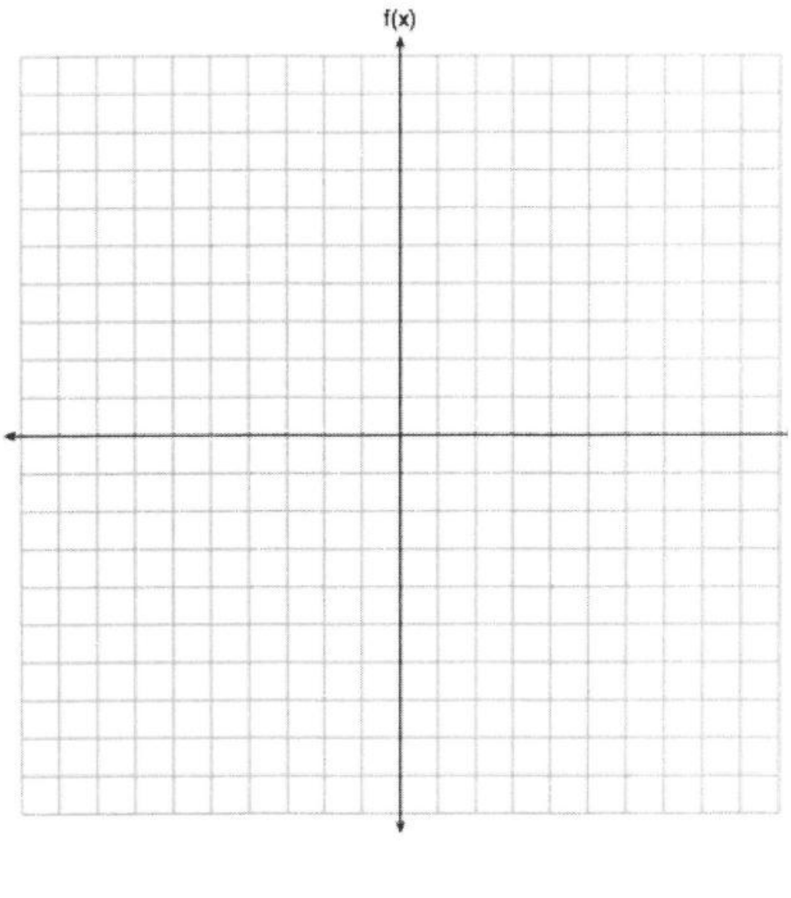
b: y-interecept

How to Graph a Linear Function:

1. Identify and plot the y-intercept, b.
2. Determine the slope, m, and use it to move around the coordinate plane.
3. Connect points with a straight line.
4. Place arrows at the ends.
5. Label the line.

Modeled Practice:

Graph the function of $f(x) = 2x + 4$



b: +4

m: 2 up
1 to the right

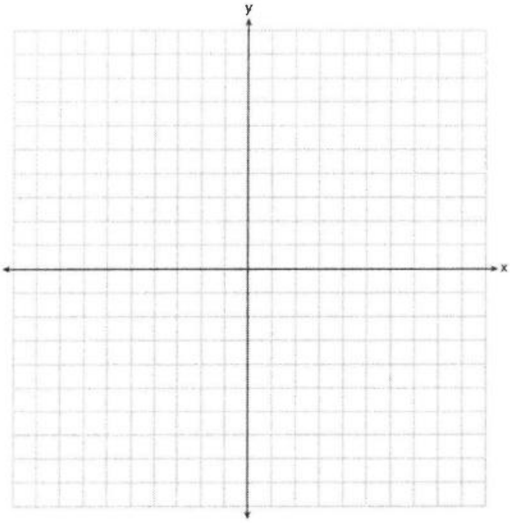
OBJECTIVE: SWBAT create the graph of a linear function.

Practice:

Create the graph of each line. Indicate the slope and y-intercept.

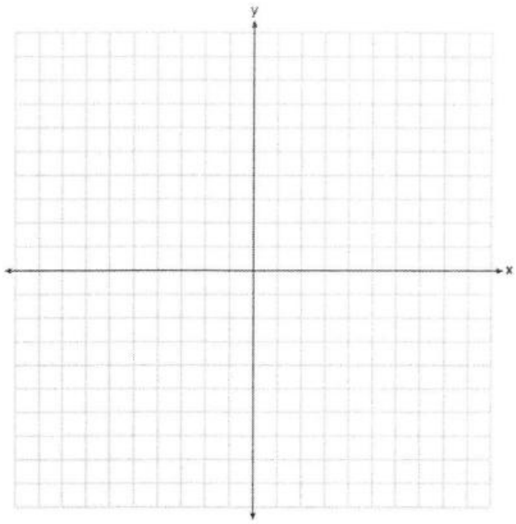
$$y = -\frac{1}{3}x + 7$$

m: b:



$$y = 3x - 6$$

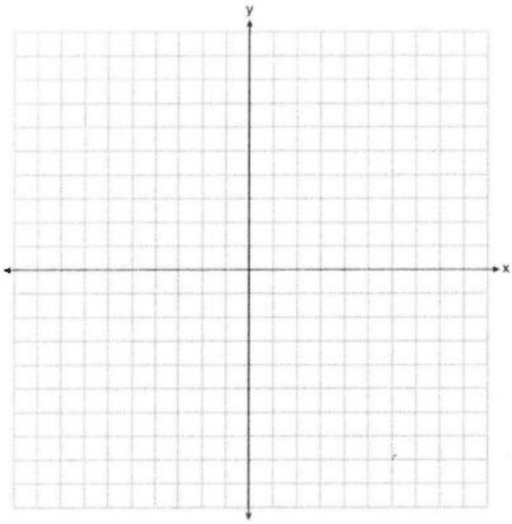
m: b:



Special Lines:

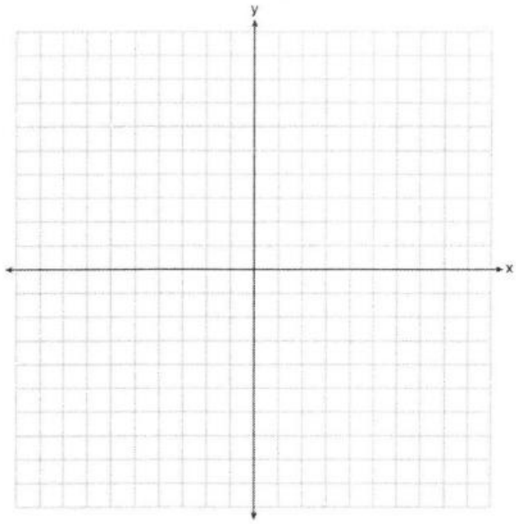
$$y = -3$$

m: b:



$$x = 4$$

m: b:



OBJECTIVE: SWBAT create the graph of a linear function.

Solving for Y First:

Use inverse operations to isolate the y.

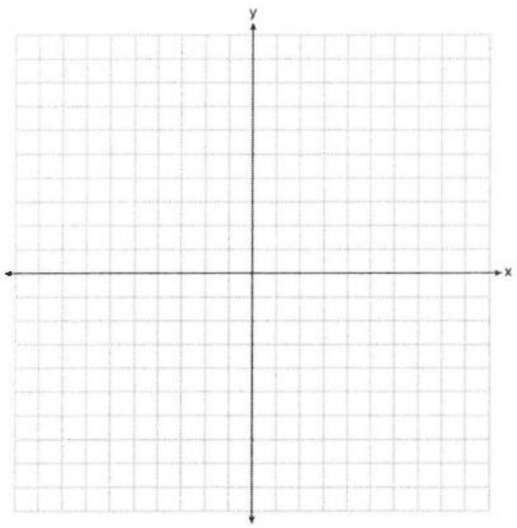
Practice:

Solve for y. Indicate the slope and y-intercept. Create the graph of each line.

A)

$$2y = -4x + 18$$

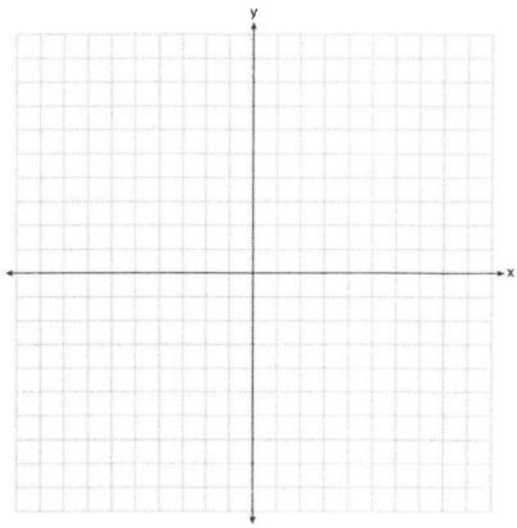
m: b:



B)

$$6x - 3y = -9$$

m: b:



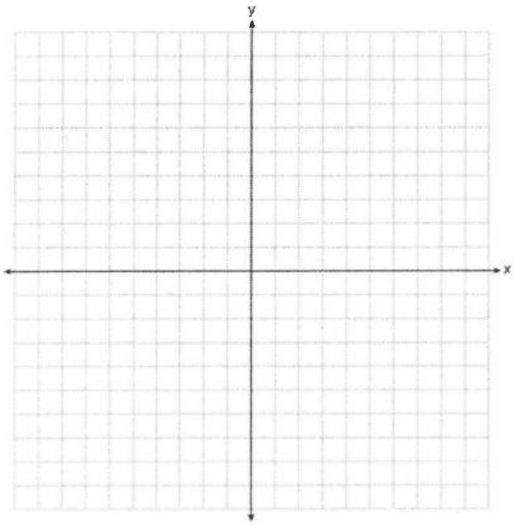
OBJECTIVE: SWBAT create the graph of a linear function.

Solve for y. Indicate the slope and y-intercept. Create the graph of each line.

C)

$$2x - 3y = 12$$

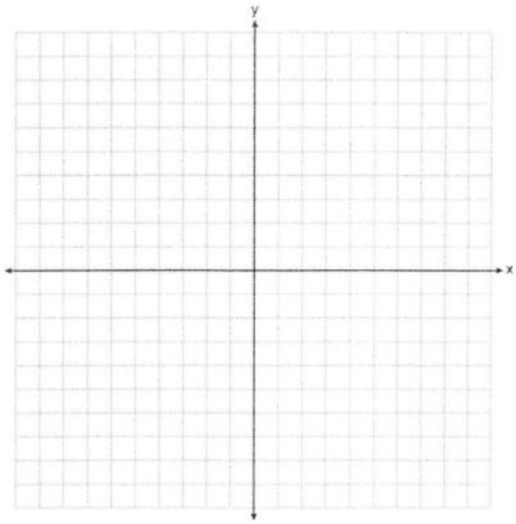
m: b:



D) *

$$-3x = -9$$

m: b:



OBJECTIVE: SWBAT create the graph of a linear function.

Closing Assessment:

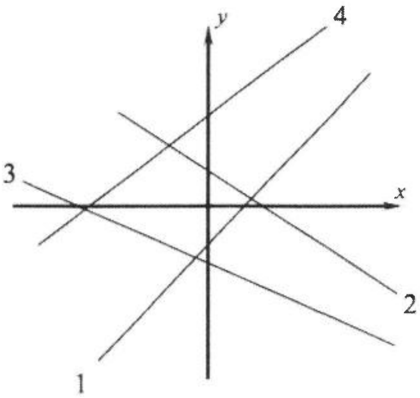
Four lines are shown graphed. Place the number of the line next to the equation that most appropriately models it.

$y = \frac{2}{3}x + 5$ ____________

$y = x - 3$ ____________

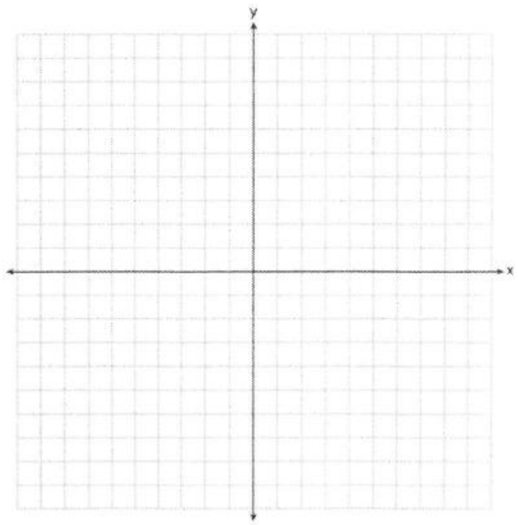
$y = -\frac{3}{4}x + 3$ ____________

$y = -\frac{1}{2}x - 4$ ____________



Solve for y. Indicate the slope and y-intercept. Create the graph of each line.

$3y + 6x = 15$ m: b:



Standard: CCSS.R-IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

OBJECTIVE: SWBAT create linear functions by translating sentences.

Name: ______________________________ Date: __________________

Lesson 18: Modeling with Linear Functions

Key Words	
Slope	**y-intercept**
• Per • Each • Monthly • Daily • Yearly • Every	• Start-up cost • One-time fee • Base fee • Base rate • Starting number

x (input)	y (output)
Goes next to our slope	Our total

Practice:

1) A satellite television company charges a one-time installation fee and a monthly service charge. The total cost is modeled by the function $y = 40 + 90x$. Which statement represents the meaning of each part of the function?

(1) y is the total cost, x is the number of months of service, \$90 is the installation fee, and \$40 is the monthly service charge.

(2) y is the total cost, x is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.

(3) x is the total cost, y is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.

(4) x is the total cost, y is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.

OBJECTIVE: SWBAT create linear functions by translating sentences.

2) The owner of a small computer repair business has one employee, who is paid an hourly rate of 522. The owner estimates his weekly profit using the function $P(x) = B600 - 22x$. In this function, x represents the number of

(1) computers repaired per week

(2) hours worked per week

(3) customers served per week

(4) days worked per week

3) The cost of airing a commercial on television is modeled by the function $C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?

(1) The commercial costs 50 to produce and 5110 per airing up to 5900

(2) The commercial costs 5110 to produce and 5900 each time it is aired.

(3) The commercial costs 5900 to produce and 5110 each time it is aired.

(4) The commercial costs 51010 to produce and can air an unlimited number of times.

4) Alex is selling tickets to a school play. An adult ticket costs 56.50 and a student ticket costs 54.00. Alex sells x adult tickets and 12 student tickets. Write a function $f(x)$, to represent how much money Alex collected from selling tickets.

5) The cost of belonging to a gym can be modeled by $C(m) = 50m + 79.50$, where $C(m)$ is the total cost for m months of membership.

State the meaning of the slope and y-intercept of this function with respect to the costs associated with the gym membership.

OBJECTIVE: SWBAT create linear functions by translating sentences.

6) Kendal bought x boxes of cookies to bring to a party. Each box contains 12 cookies. She decides to keep two boxes for herself. She brings 60 cookies to the party. Which equation can be used to find the number of boxes, x, Kendal bought?

(1) $2x - 12 = 60$

(2) $12x - 2 = 60$

(3) $12x - 24 = 60$

(4) $24 - 12x = 60$

7) A car leaves Albany, NY, and travels west toward Buffalo, NY. The equation $D = 280 - 59t$ can be used to represent the distance, D, from Buffalo after t hours. In this equation, the 59 represents the

(1) car's distance from Albany

(2) speed of the car

(3) distance between Buffalo and Albany

(4) number of hours driving.

8) Sandy programmed a website's checkout process with an equation to calculate the amount customers will be charged when they download songs.

The website offers a discount. If one song is bought at the full price of 51.29, then each additional song is 50.99.

State an equation that represents the cost, C, when s songs are downloaded.

OBJECTIVE: SWBAT create linear functions by translating sentences.

Closing Assessment:

A plumber has a set fee for a house call and charges by the hour for repairs. The total cost of her services can be modeled by $c(t) = 125t + 95$. Fill in the blanks with either $c(t)$, 125, t, or 95 based on the description.

The flat fee is ______________

The number of hours of repair is ____________

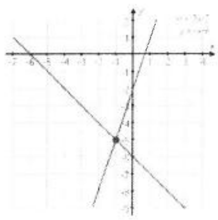
The total cost is ____________

The cost per hour is _______________

Vocabulary:

Solution – A value we can put in place of a variable to make the equation true.

Solution to a System – the point of intersection that lies on both lines.



The solution to a system of equations is where their lines intersect/cross.

A **system of linear equations** is a set of equations of straight lines. When working with a system of equations, we are working with two equations at the same time.

Solving Systems of Linear Equations Graphically

The goal of solving systems of lienar equations graphically, is to graph both lines and locate the point of intersection.

There are three possible solutions; one solution, no solution, or infinite solutions.

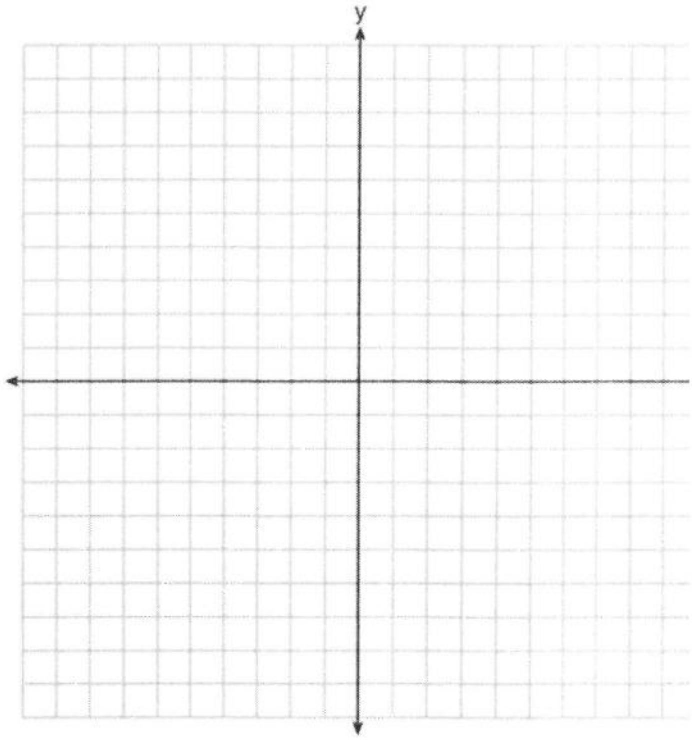
OBJECTIVE: SWBAT create linear functions by translating sentences.

One Solution

Solve the system of equations below graphically.

$y = -x + 7$

$y = 2x + 1$



We can see exactly 1 point of intersection.

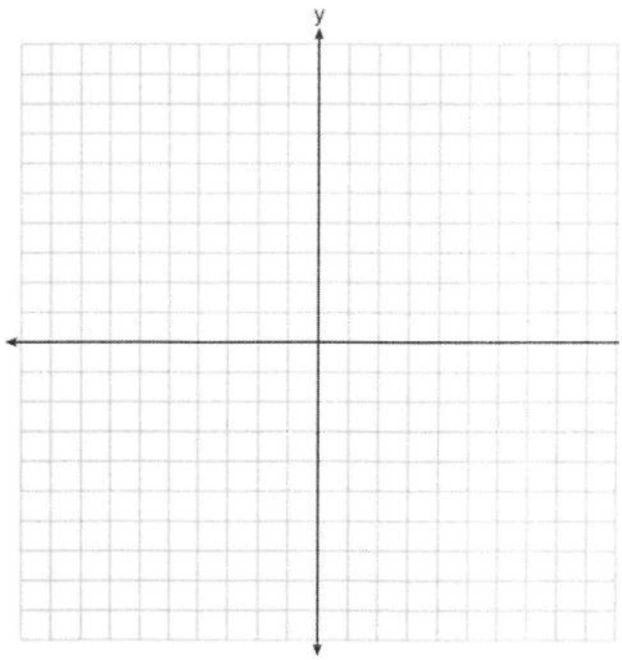
We have one solution, _______________.

No Solution

Solve the following system of equations graphically:

$y = 2x + 4$

$y = 2x - 2$



We can see no points of intersection; these lines are parallel. No solution!

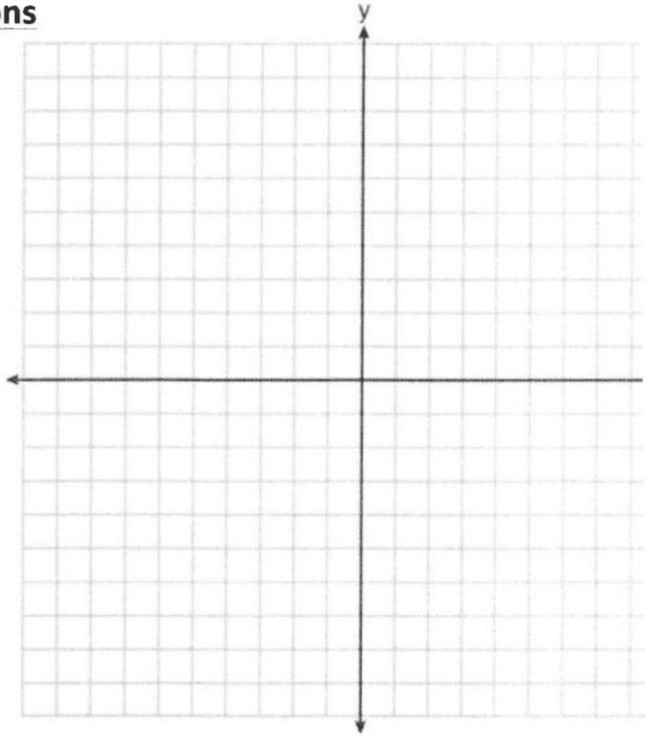
OBJECTIVE: SWBAT create linear functions by translating sentences.

Infinite Solutions

Solve the following system of equations graphically:

$y = -2x + 1$

$2x + y = 1$

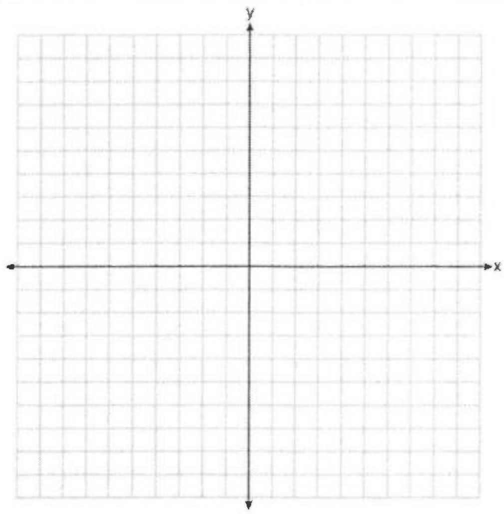


These are the **same lines** that lie directly on top of one another. Infinite solutions!

Independent Practice:

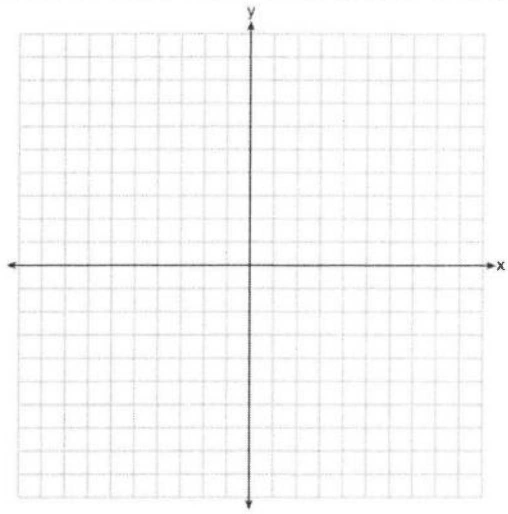
Solve each of the following systems of equations graphically:

a) $y = x + 6$
$y = -x + 2$

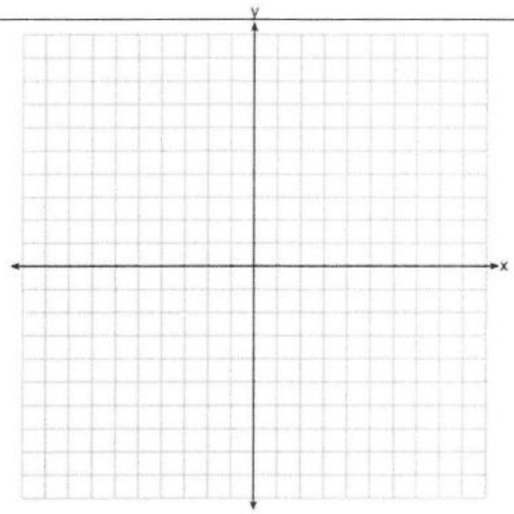


OBJECTIVE: SWBAT create linear functions by translating sentences.

b) $y = -x$
$2x + y = 3$



c) $y = 4x + 3$
$y = -x - 2$



Closing Assessment:

Using your calculator, find the solution for:

$y = -6x - 3$ $y = -x + 2$ Solution = ___________

Standards: CCSS.MATH.CONTENT.HSA.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. CCSS.MATH.CONTENT.HSA.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

OBJECTIVE: SWBAT represent data on a scatter plot and create linear regression models.

Name: ______________________________ Date: ______________

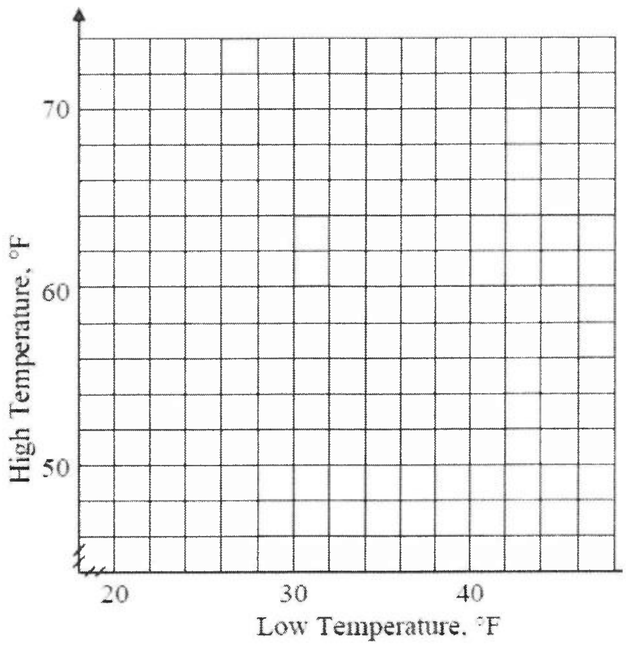
Lesson 19: Linear Regression

Scatter Plots:

A survey was taken of 10 low and high temperatures, in Fahrenheit, in the month of April to try to establish a relationship between a day's low temperature and high temperatures.

Low Temperature, x	26	28	30	32	34	35	37	38	41	45
High Temperature, y	49	50	57	54	60	58	64	66	63	72

a) Construct a scatter plot of this bivariate data set on the grid



b) Draw a line of best fit through this data.
(Draw a line through the middle of the points)

c) If the low temperate is 20, what will be the predicted high temperature?

Regression

The term **regression** refers to the process of finding the best fit for data. Regression allows us to come up with an equation that would be fit the data so we can predict future data!

OBJECTIVE: SWBAT represent data on a scatter plot and create linear regression models.

Practice:

Ex 1) The table below shows the number of grams of carbohydrates, x, and the number of Calories, y, of six different foods.

Carbohydrates (x)	Calories (y)
8	120
9.5	138
10	147
6	88
7	108
4	62

Which equation best represents the line of best fit for this set of data?

(1) $y = 15x$

(2) $y = 0.07x$

(3) $y = 0.1x - 0.4$

(4) $y = 14.1x + 5.8$

Steps

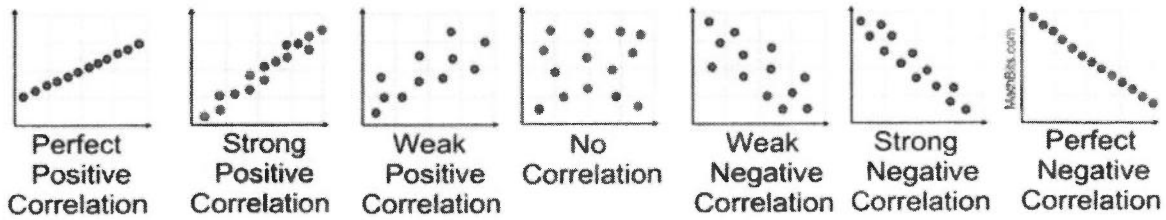
1. Make sure `STAT DIAGNOSTICS` and `STAT PLOT` are turned on
2. Type all necessary data in L_1 and L_2.
   L_1 will be our x values (independent variable)
   L_2 will be our y values (dependent variable)
3. Press `ZOOM 9` to view the scatter plot.
4. If the scatter plot resembles a linear line, use linear regression
5. Press `STAT→CALC→4 "LinReg(ax+b)`
6. Press `ENTER`
7. Substitute values in for a and b to create your linear equation.

Correlation Coefficient

To determine if this is a "good fit" for our data, we look at the **correlation coefficient, r.**

If r is close to 1 or -1, the model is considered a good fit.

A correlation larger than ± 0.80 is considered stong, whereas a correlation less than ± 0.50 is considered weak.



Correlation does not imply causation! A **causal relation** between two events exists if the first event, the cause, causes the second event, the effect. For example, if we turn the volume up on a radio, it will be louder.

OBJECTIVE: SWBAT represent data on a scatter plot and create linear regression models.

Ex 2) Erica, the manager at Stellarbeans, collected data on the daily high temperature and revenue from coffee sales. Data from nine days this past fall are shown in the table below.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9
High Temperature, t	54	50	62	67	70	58	52	46	48
Coffee Sales, f(t)	$2900	$3080	$2500	$2380	$2200	$2700	$3000	$3620	$3720

State the linear regression function, $f(t)$, that estimates the day's coffee sales with a high temperature of t. Round all values to the *nearest integer*.

State the correlation coefficient, r, of the data to the *nearest hundredth*.

Does r indicate a strong linear relationship between the variables? Explain your reasoning.

OBJECTIVE: SWBAT represent data on a scatter plot and create linear regression models.

Using Linear Regression to Predict Data

Ex 3) The data table below shows the median diameter of grains of sand and the slope of the beach for 9 naturally occurring ocean beaches.

Median Diameter of Grains of Sand, in Millimeters (x)	0.17	0.19	0.22	0.235	0.235	0.3	0.35	0.42	0.85
Slope of Beach, in Degrees (y)	0.63	0.7	0.82	0.88	1.15	1.5	4.4	7.3	11.3

Write the linear regression equation for this set of data, rounding all values to the *nearest thousandth.*

Using this equation, predict the slope of a beach, to the *nearest tenth of a degree*, on a beach with grains of sand having a median diameter of 0.65 mm.

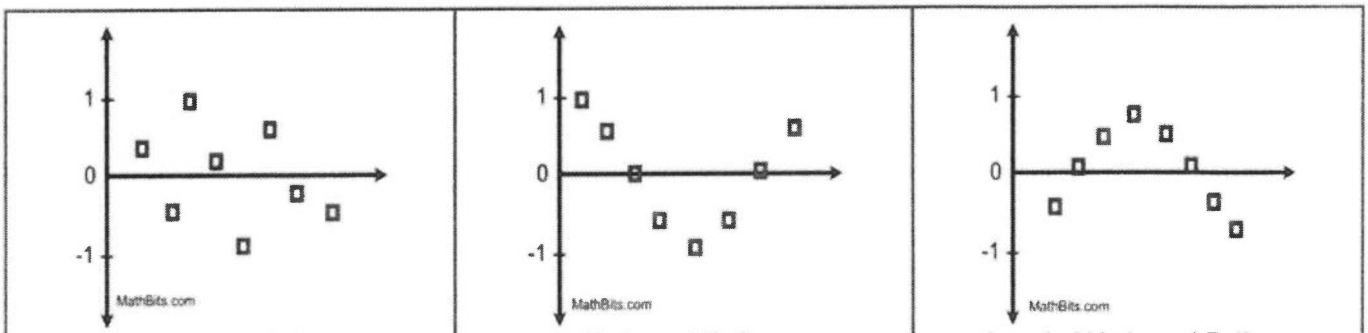
We simply substitute our value for the median diameter of sand into x, then solve for y!

Residuals

Residuals help to determine if a line of best fit is appropriate for the data.

If our residual plot follows **no pattern**, then the data **IS linear**

If our residual plot follows **a pattern**, then it is **NOT linear.**

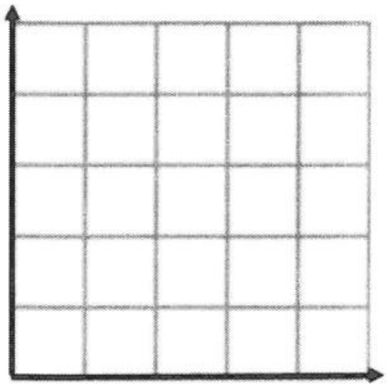


Random, No Pattern "Linear Appropriate" | U-shaped Pattern "Non-Linear Appropriate" | Inverted U-shaped Pattern "Non-Linear Appropriate"

OBJECTIVE: SWBAT represent data on a scatter plot and create linear regression models.

Practice:

Ex 1) Make a scatter plot for the table of values below.



x	1	2	3	4	5
y	2	1	3.5	3	4.5

a) Using your graphing calculator, find the equation for the line of best fit.

b) Type the line of best fit $y = 0.7x + 0.7$ into Y= and copy the predicted y values into the table below.

x	1	2	3	4	5
y	1.4				

c) Subtract the **actual y values − the predicted y values.** Fill these differences in the table below.

x	1	2	3	4	5
$actual - predicted$ Residuals	0.6	−1.1			

d) Plot these points $(x, residual)$ on the same scatter plot with square shapes.

e) Does your residual plot follow a pattern? Is a linear relationship a good fit?

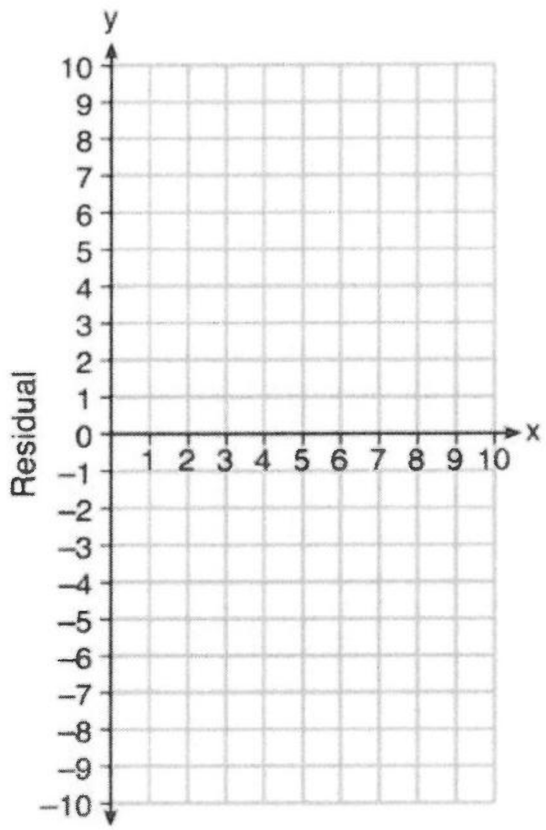
OBJECTIVE: SWBAT represent data on a scatter plot and create linear regression models.

Ex 2) The table below shows the raw test score based on the hours tutored.

Tutor Hours, x	Raw Test Score	Residual (Actual – Predicted)
1	30	1.3
2	37	1.9
3	35	–6.4
4	47	–0.7
5	56	2.0
6	67	6.6
7	62	–4.7

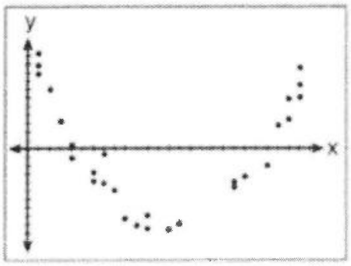
a) Use the data below to write the linear regression equation for the raw test score based on the hours tutored. Round all values to the *nearest hundredth*.

b) Create a residual plot on the axes below, using the residual scores in the table above. Then based on the residual plot, state whether the equation is a good fit for the data. Justify your answer.



OBJECTIVE: SWBAT represent data on a scatter plot and create linear regression models.

Ex 3) After performing analyses on a set of data, Jackie examined the scatter plot of the residual values for each analysis. Which scatter plot indicates the best linear fit for the data?

A

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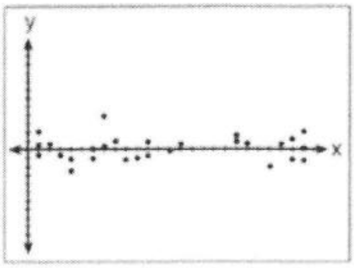
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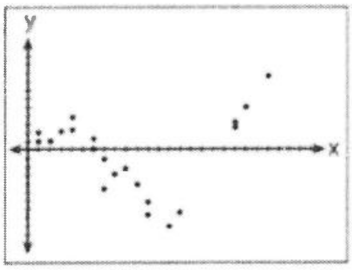
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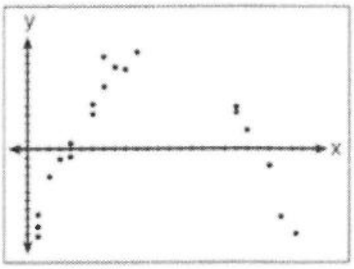
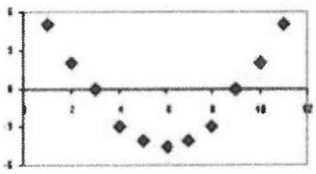
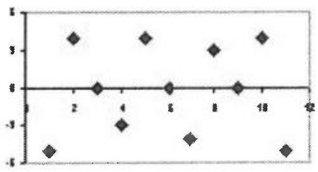
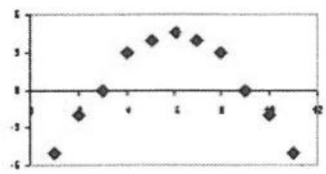


Closing Assessment:

Which of the following residual plots indicate a good fit for a linear model?

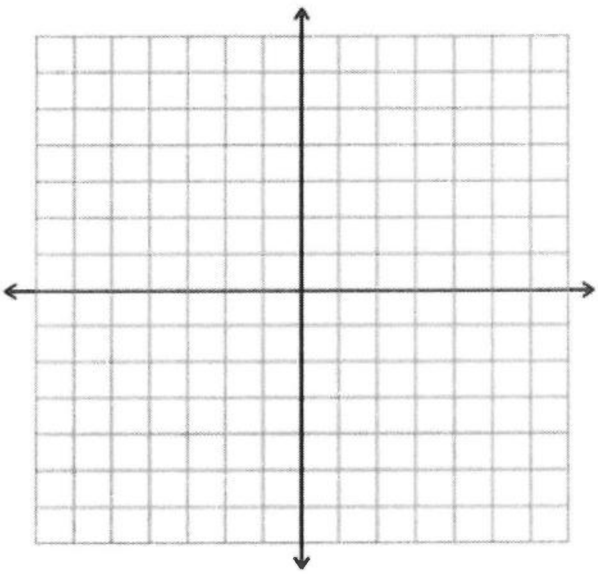
Explain your choice based on what you learned about residuals.

OBJECTIVE: SWBAT create the graph of a linear inequality.

Name: ______________________________ Date: __________________

Lesson 20: Graphing Linear Inequalities

Graphing a linear inequality is just an extension of graphing a straight line!

Solution	Steps
Ex.) On the set of axes below graph the inequality $y \leq -\frac{2}{3}x + 2$ 	1. Solve for y 2. Graph the points of the line. 3. Determine the line: Solid: $\leq$ or $\geq$ Dashed: $<$ or $>$ 4. Shade appropriately in the direction where the inequality statement is true. Above: $>$ or $\geq$ Below: $<$ or $\leq$ 5. Arrows and label.

****All of the points in the shaded area and on a solid line are part of the solution set.****

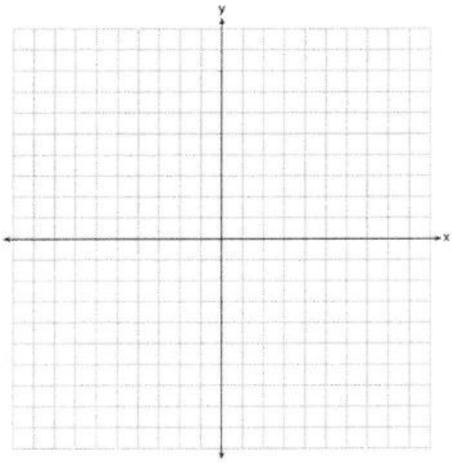
OBJECTIVE: SWBAT create the graph of a linear inequality.

Practice:

Graph each of the following linear inequalities. Circle whether each line will be dashed or solid AND whether we will be shading above or below the line.

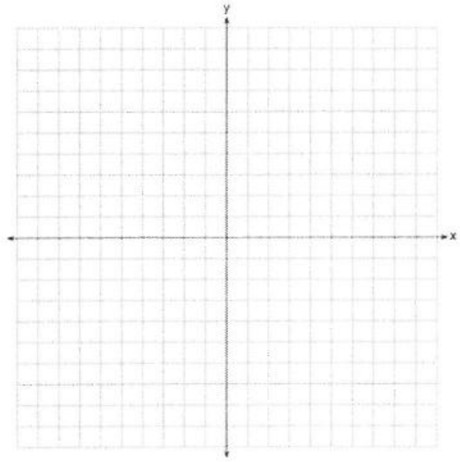
A) $y < \frac{2}{3}x + 8$

Line: DASHED/SOLID
Shade: ABOVE/BELOW



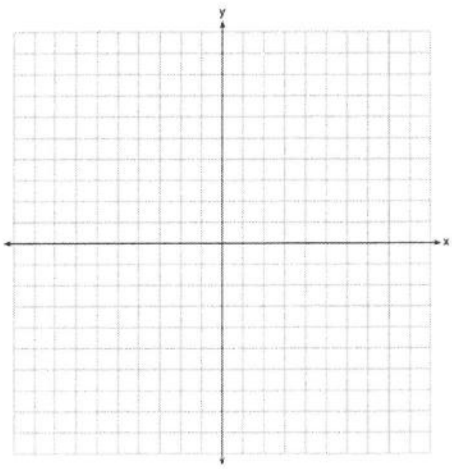
B) $y \geq -3x + 9$

Line: DASHED/SOLID
Shade: ABOVE/BELOW



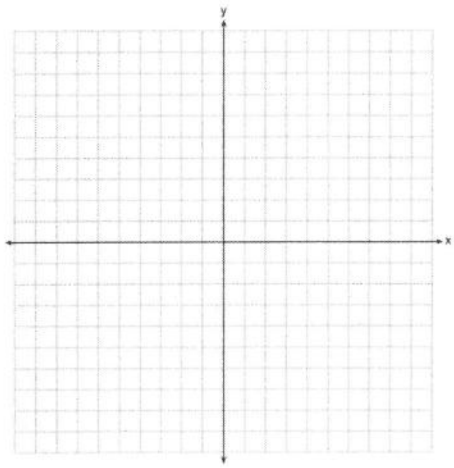
C) $x < -2$

Line: DASHED/SOLID
Shade: ABOVE/BELOW



D) $y \geq 3$

Line: DASHED/SOLID
Shade: ABOVE/BELOW

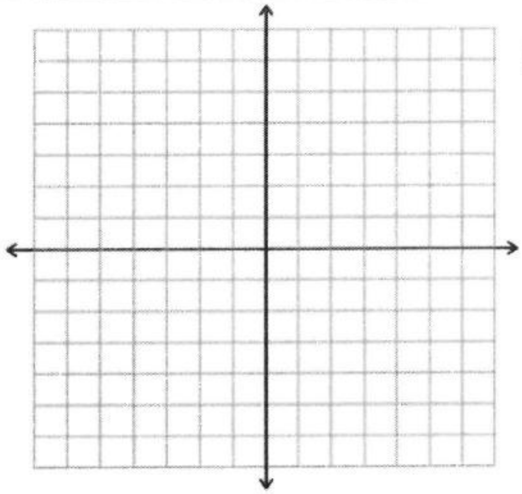


OBJECTIVE: SWBAT create the graph of a linear inequality.

To graph a linear inequality that is not in $y = mx + b$ form, first we must solve for y.
Remember, when we multiply or divide by a negative value, we must flip the inequality!

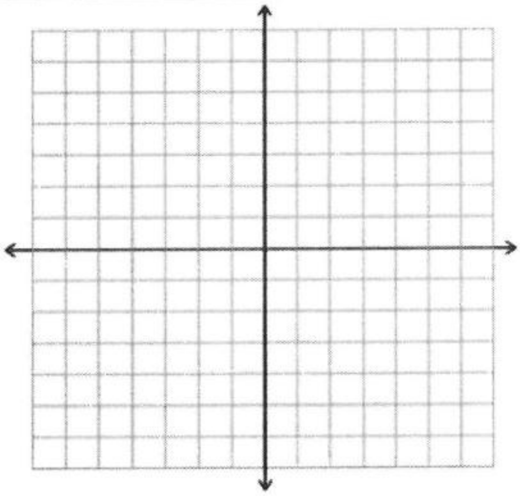
Graph each of the following linear inequalities. Then state a point in the solution set.

a) $y + 3x > 4$



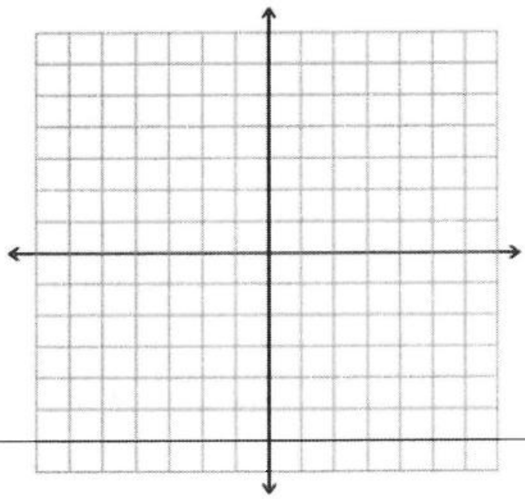
Point in the solution set

b) $-3y > 2x - 15$



Point in the solution set

c) $-3y > 2x - 15$

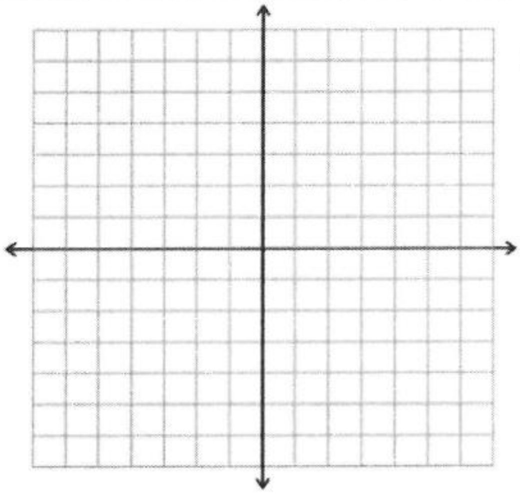


Point in the solution set

OBJECTIVE: SWBAT create the graph of a linear inequality.

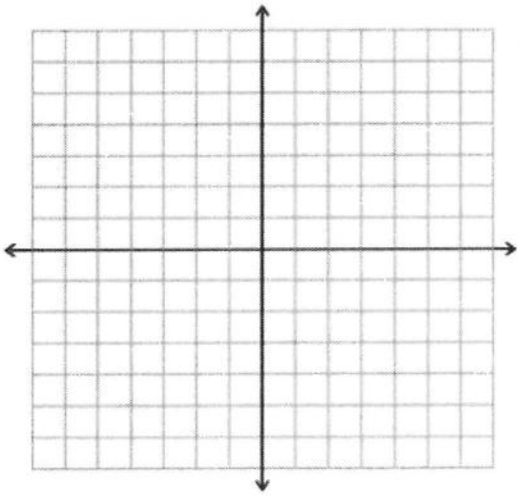
Graph the following linear inequalities and state a point in the solution set.

1) $2y > -6x + 10$



Point in the solution set

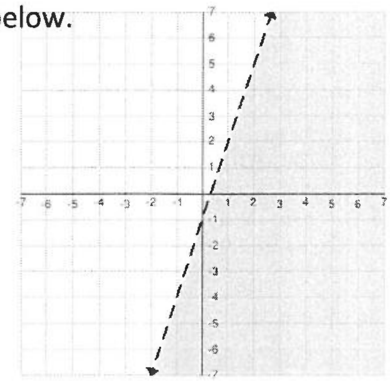
2) $4 - 3y \geq 2x$



Point in the solution set

Closing Assessment:

Brad graphed the linear inequality $y > 3x - 1$ as shown in the graph below. Is the point $(1, 3)$ in the solution set? Explain your answer.



Standards: CCSS.R-IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. CCSS.8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. CCSS.A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Standards: CCSS.R-IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. CCSS.8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. CCSS.A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

OBJECTIVE: SWBAT create a system of inequalities to solve a word problem.

Name: ______________________ Date: ______________

Lesson 21: Modeling with Linear Inequalities

Translating Inequalities:

$<$	$>$
Is less than Is smaller than Is fewer than Below	Is greater than Is larger than Is more than Above
$\leq$	$\geq$
Maximum At most Is not greater than Is not more than	Minimum At least Is not less than Is not smaller than

Modeling Linear Inequalities:

1)

Joy wants to buy strawberries and raspberries to bring to a party. Strawberries cost \$1.60 per pound and raspberries cost \$1.75 per pound. If she only has \$10 to spend on berries, which inequality represents the situation where she buys x pounds of strawberries and y pounds of raspberries?

1) $1.60x + 1.75y \leq 10$
2) $1.60x + 1.75y \geq 10$
3) $1.75x + 1.60y \leq 10$
4) $1.75x + 1.60y \geq 10$

OBJECTIVE: SWBAT create a system of inequalities to solve a word problem.

2)

The math department needs to buy new textbooks and laptops for the computer science classroom. The textbooks cost $116.00 each, and the laptops cost $439.00 each. If the math department has $6500 to spend and purchases 30 textbooks, how many laptops can they buy?

3)

David has two jobs. He earns $8 per hour babysitting his neighbor's children and he earns $11 per hour working at the coffee shop. Write an inequality to represent the number of hours, x, babysitting and the number of hours, y, working at the coffee shop that David will need to work to earn a minimum of $200. David worked 15 hours at the coffee shop. Use the inequality to find the number of full hours he must babysit to reach his goal of $200.

OBJECTIVE: SWBAT create a system of inequalities to solve a word problem.

4)

Sarah wants to buy a snowboard that has a total cost of $580, including tax. She has already saved $135 for it. At the end of each week, she is paid $96 for babysitting and is going to save three-quarters of that for the snowboard. Write an inequality that can be used to determine the minimum number of weeks Sarah needs to babysit to have enough money to purchase the snowboard. Determine and state the minimum number of full weeks Sarah needs to babysit to have enough money to purchase this snowboard.

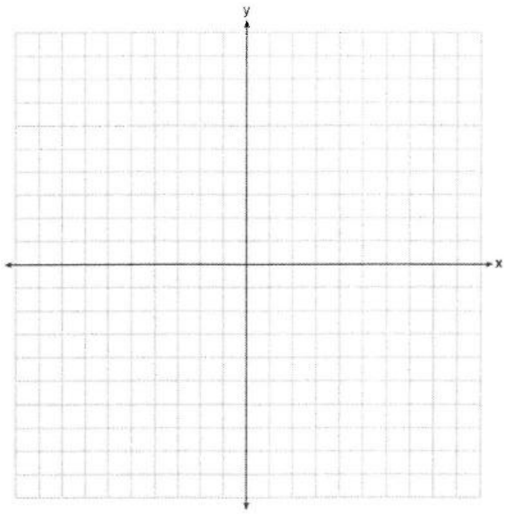
OBJECTIVE: SWBAT create a system of inequalities to solve a word problem.

Systems of linear inequalities are composed of two separate linear inequalities.
Graphing a system of linear inequalites involves graphing the two separate linear inequalites on the same axes, and looking at the sections of the graphs **shading that overlap one another.**

How to Graph a System of Inequalities:

1. Graph the first inequality.
   a) Solve the equation for y.
   b) Identify the y-intercept, b.
   c) Use the slope, m, to plot more coordinate points.
   d) Determine the type of line.
   e) Determine where to shade.
   f) Place arrows at the ends.
   g) Label the line.
2. Graph the second inequality.
3. Identify the overlapping section and label it with an *S*.

$x + 2y \leq 8$ $\qquad$ $y \leq x + 4$



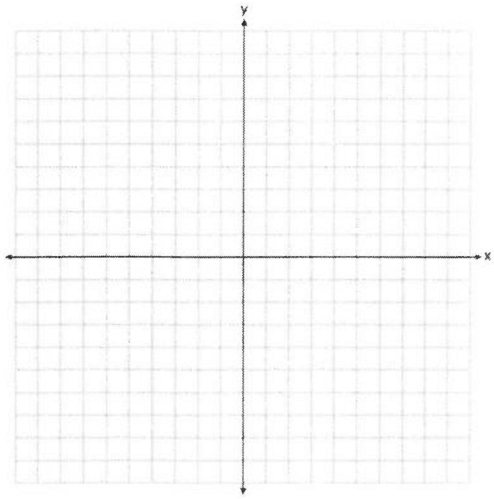
Is the point (-3, 9) in the solution set? Why or why not?

OBJECTIVE: SWBAT create a system of inequalities to solve a word problem.

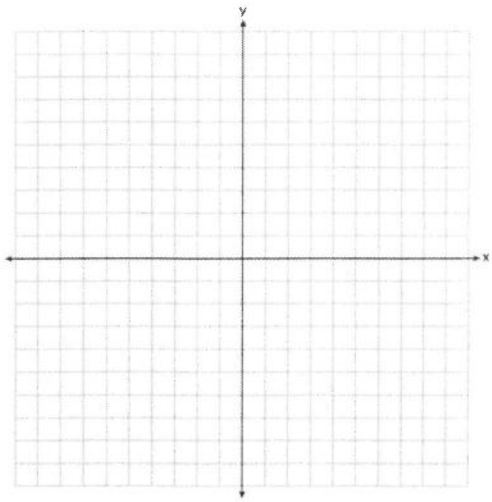
Practice:

Solve each of the following systems of linear inequalities graphically.

$y \geq \frac{3}{4}x - 3$ $y < 5x$

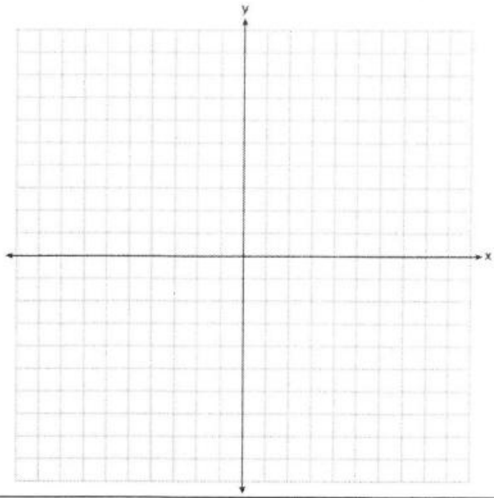


$y - 2 < 0$ $y \geq x + 4$

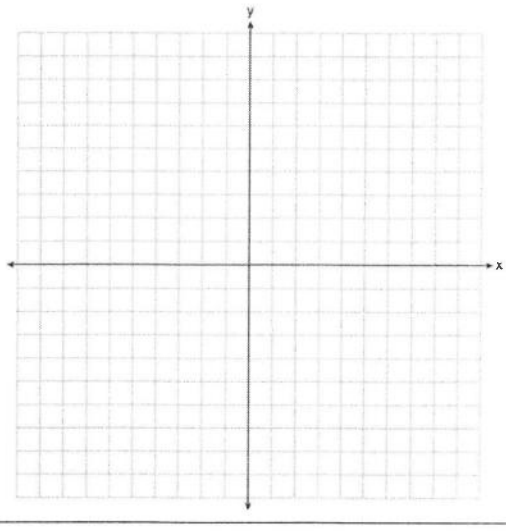


OBJECTIVE: SWBAT create a system of inequalities to solve a word problem.

$y + 3x \leq -5$ $y \geq 1$



$5y - 2x < -25$ $y < -x + 2$



OBJECTIVE: SWBAT create a system of inequalities to solve a word problem.

Closing Assessment:

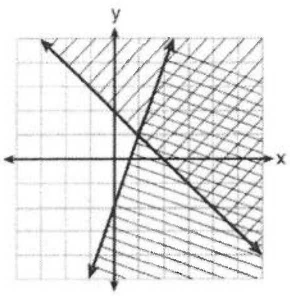
Answer the questions based on systems of linear inequalities.

1) Given: $y + x > 2$

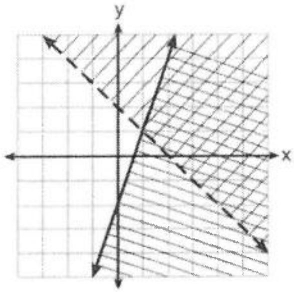
$y \leq 3x - 2$

Which graph shows the solution of the given set of inequalities?

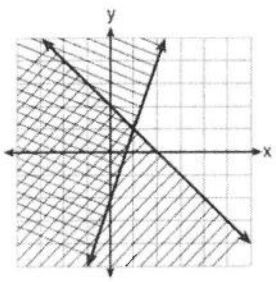
a)



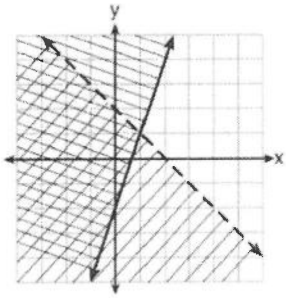
b)



c)

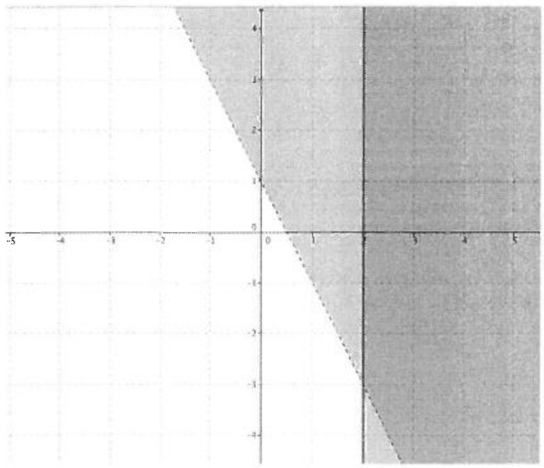


d)



2) The following system of inequalities is graphed below:

$$\begin{cases} x \geq 2 \\ y > -2x+1 \end{cases}$$



Circle all ordered pairs that are solutions to the system of inequalities. Explain.

(a) $(0,1)$ (c) $(3,-2)$

(b) $(2,0)$ (d) $(2,-3)$

Standards: CCSS.Math.Content.HSA-CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. CCSS.Math.Content.HSA-REI.D.10: Understand that the graph of an inequality in two variables is the set of all its solutions plotted in the coordinate plane. CCSS.Math.Content.HSA-REI.D.12: Graph the solutions to a linear inequality in two variables. CCSS.A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

OBJECTIVE: SWBAT use substitution to solve a system of equations.

Name: ______________________________ Date: ________________

Lesson 22: Solve Systems Algebraically

How To Solve Systems through Substitution:

The goal of this method is the replace one of the equations with and equivalent expression by solving for one variable in one of the equations.

Solution	Steps
$y = x + 7$ $y = -2x + 1$	**1.** Solve one (or both) equation for one of the variables (we want it to say $x =$ or $y =$) **2.** Substitute the expression for the solved variable in the **other** equation (or set equations equal).
$x + 7 = -2x + 1$ $+2x \quad +2x$ $3x + 7 = 1$ $-7 \quad -7$ $\frac{3x}{3} = \frac{-6}{3}$ $x = -2$	**3.** Solve for the other variable
$y = (-2) + 7$ $y = 5$	**4.** Substitute the value of the solved variable into either equation.
Solution: $(-2, 5)$	**5.** Write your solution as an ordered pair (x, y)

OBJECTIVE: SWBAT use substitution to solve a system of equations.

Practice:

Solve each of the following systems of equations algebraically using substitution.

a) $y = -5x - 17$ $y = -x - 1$	**b)** $y = 6x - 11$ $-2x - 3y = -7$
c) $y = -x + 3$ $3y + x = 5$	**d)** $2x - 3y = -1$ $x = y + 1$

OBJECTIVE: SWBAT use substitution to solve a system of equations.

How To Solve Systems through Elimination:

The goal of the elimination method is to create a situation where one set of variables will cancel each other out when the equations are added together.
It may be necessary in this method to create coefficients for these variables to make the cancellation possible

Solution	Steps
$x + y = 12$ $2x - y = -6$ $x + \cancel{y} = 12$ $2x - \cancel{y} = -6$ $3x = 6$	**1.** Choose a variable to eliminate. **2.** Add the equations together, which will cause this variable to cancel.
$\frac{3x}{3} = \frac{6}{3}$ $x = 2$	**3.** Solve for the remaining variable.
$(2) + y = 12$ $-2 \qquad -2$ $y = 10$	**4.** Substitute this variable into either original equation and solve for the other variable.
Solution: $(2, 10)$	**5.** Write your solution as an ordered pair.

OBJECTIVE: SWBAT use substitution to solve a system of equations.

Guided Practice:

Steps	Using *addition* to eliminate a variable:	Using *subtraction* to eliminate a variable:
1) Line up the variables.	$x + y = 8$ $x - y = -6$	$x - 2y = 14$ $x + 3y = 9$
2) Decide which variable ("x" or "y") will be *easier* to eliminate (same coefficient).		
3) If the coefficients are the same sign, subtract. If the coefficients are inverses, add.		
4) Solve the equation for the other variable ("y" or "x").		
5) Substitute the answer into one of the original equations to get the other variable.		
6) Write the answer as an ordered pair (x, y).		
7) Check your solution in BOTH original equations.		

Quick Check:

Given the following system of linear equations.

$$x + 2y = -2$$
$$3x - 2y = 10$$

Which variable should be eliminated? __________

OBJECTIVE: SWBAT use substitution to solve a system of equations.

Systems through Elimination Using Multiplication:

Sometimes we have to multiply by numbers to create inverse coefficients. Then, we can add or subtract to eliminate the variable.

Ask Yourself:

1. Are the variables lined up?
2. Use the flip and multiply method. (Flip the first variables' coefficients and distribute).
3. Do they have the same sign?
    i. If yes, subtract.
    ii. If no, add.
4. Then, solve for the variable left over.
5. Next, substitute the solution in Step 4 to solve for the other variable.

Practice:

Use the flip and multiply method to eliminate a variable.

$4x + 5y = 12$
$-2x + y = 8$

OBJECTIVE: SWBAT use substitution to solve a system of equations.

Practice:

Solve each of the following systems of equations algebraically using elimination.

a) $x + 2y = 9$ $-x + y = -3$	**b)** $x + 3y = 13$ $x + y = 5$
c) $3x + 2y = 12$ $5x - 2y = 4$	**d)** $10x - 10y = 30$ $5x - 3y = 9$

OBJECTIVE: SWBAT use substitution to solve a system of equations.

Closing Assessment:

What is the value of a in the system below?

$$a + 3b = 13$$
$$-3a - 3b = -15$$

(1) 1

(2) 7

(3) 4.5

(4) 4

Standard: CCSS.MATH.CONTENT.HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. CCSS.MATH.CONTENT.HSA.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.: CCSS.Math.Content.HSA-SSE.A.1a: Interpret aspects of an expression (i.e. terms, factors, and coefficients). CCSS.Math.Content.HSA-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. CCSS.Math.Content.HSA-CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. CCSS.Math.Content.HSA-REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. CCSS.Math.Content.HSA-REI.C.6: Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. Standard: CCSS.MATH.CONTENT.HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. CCSS.MATH.CONTENT.HSA.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

OBJECTIVE: SWBAT create a system of equations based on a real-world problem and then solve.

Name: ______________________ Date: ______________

Lesson 23: Modeling with Systems

How to Create Systems of Linear Equations:

1. Identify both of the unknown quantities with a variable (let statement).
2. Create two equations using the variables you made in your let statement.
3. Solve your system algebraically.

***Exercise* #1:** Jonathan has nine bills in his wallet that are all either five-dollar bills or ten-dollar bills.

(a) Fill out the following table to see the dependence of the two variables and how they then determine how much money Jonathan has.

Number of fives, f	Number of tens, t	Amount of Money, \$
0	9	$=9(10)=\$90$
1	8	$=1(5)+8(10)=\$85$
2	7	$=2(5)+7(10)=\$80$
3	6	$=3(5)+6(10)=\$75$

(b) If f represents the number of \$5 bills and t represents the number of \$10 bills, then what does the following expression calculate? Explain.

$$5f+10t$$

Since f represents the number of \$5 bills, the term $5f$ represents the amount of dollars we have in \$5 bills. Likewise, the $10t$ term represents the amount of dollars we have in \$10 bills. When we add them together, we have the total amount of dollars based on the number of fives, f, and the number of tens, t.

(c) If Jonathan has a total of \$55, set up a system of equations involving f and t that could be used to determine how many of each bill he has. Solve the system. Remember that he has 9 total bills.

$f+t=9$

$5f+10t=55$

The first equation comes from the fact that if we add the number of fives we have, f, to the number of tens we have, t, we have a total of 9 bills. The second equation is simply our work from (b) set equal to \$55.

Multiply the first equation by -5:

$-5(f+t)=-5(9)$

$-5f-5t=-45$

Now add to the 2nd :

$-5f-5t+5f+10t=-45+55$

$5t=10 \Rightarrow \frac{5t}{5}=\frac{10}{5} \Rightarrow t=2$

$f+2=9 \Rightarrow f=7$

There are 2 tens and 7 fives.

(d) Let's say that we were told that Jonathan had seven bills that were all 5's and 20's and we were also told that he had a total of \$120. Set up and solve a system to help evaluate whether we could have been told true information.

Let f = the number of 5's

Let w = the number of 20's

$f+w=7$

$5f+20w=120$

Multiply the 1st equation by -5:

$-5(f+w)=-5(7)$

$-5f-5w=-35$

Now add and eliminate!

$-5f-5w+5f+20w=-35+120$

$15w=85 \Rightarrow \frac{15w}{15}=\frac{85}{15}$

$w=5.\overline{6}$

We could NOT have been told true information. The information given would lead us to conclude that Jonathan had 5.66 twenty dollar bills. But, you cannot have a fraction

OBJECTIVE: SWBAT create a system of equations based on a real-world problem and then solve.

Practice:

1) Jacob and Zachary go to the movie theatre and purchase refreshments for their friends. Jacob spends a total of \$18.25 on two bags of popcorn and three drinks. Zachary spends a total of \$27.50 for four bags of popcorn and two drinks. Determine and state the price of a bag of popcorn and the price of a drink.

2) The local deli charges a fee for delivery. On Monday, they delivered two dozen bagels to an office at a total cost of \$8. On Tuesday, three dozen bagels were delivered at a total cost of \$11. Write and solve a system of equations that could be used to find the cost of a dozen bagels b, if the delivery fee is f.

OBJECTIVE: SWBAT create a system of equations based on a real-world problem and then solve.

Applications:

1) If not given variables, write 'let' statements for each variable.
2) Write inequalities based on the given variables.
3) Solve the inequalities for the dependent (y) variable.
4) Graph the inequalities.
5) Determine the solution set.

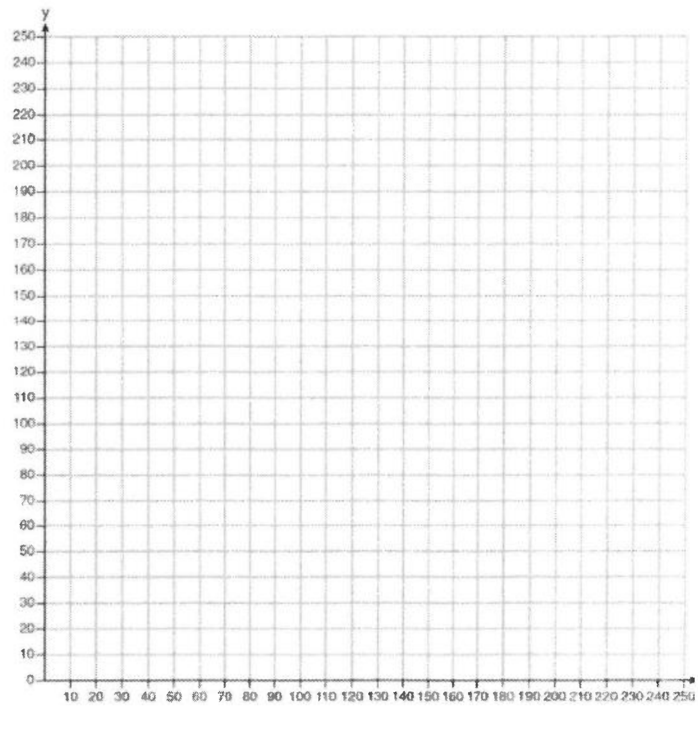
Practice:

The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost $12.50 and child tickets cost $6.25. The cinema's goal is to sell at least $1500 worth of tickets for the theater.

(a) Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, x, and child tickets, y, that would satisfy the cinema's goal.

(b) Graph the solution to this system of inequalities on the set of axes below. Label the solution with an S.

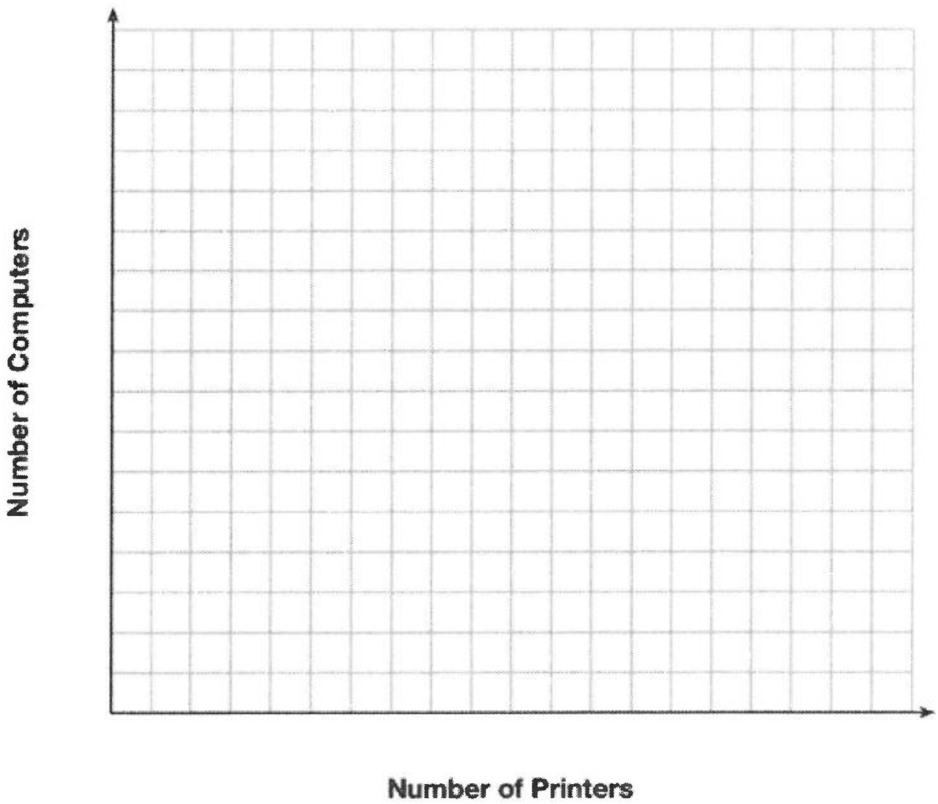
(c) Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn.



OBJECTIVE: SWBAT create a system of equations based on a real-world problem and then solve.

An online electronics store must sell at least $2500 worth of printers and computers per day. Each printer costs $50 and each computer costs $500. The store can ship a maximum of 15 items per day.

(a) On the set of axes below, write and graph a system of inequalities that models these constraints.



(b) Determine a combination of printers and computers that would allow the electronics store to meet all of the constraints. Explain how you obtained your answer.

Standards: CCSS Math.Content.HSA-CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. CCSS.Math.Content.HSA-REI.D.10: Understand that the graph of an inequality in two variables is the set of all its solutions plotted in the coordinate plane. CCSS.Math.Content.HSA-REI.D.12: Graph the solutions to a linear inequality in two variables. CCSS.A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. CCSS.MATH.CONTENT.HSA.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. CCSS.MATH.CONTENT.HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. Standard: CCSS.MATH.CONTENT.HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. CCSS.MATH.CONTENT.HSA.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

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Name: ______________________________ Date: __________________

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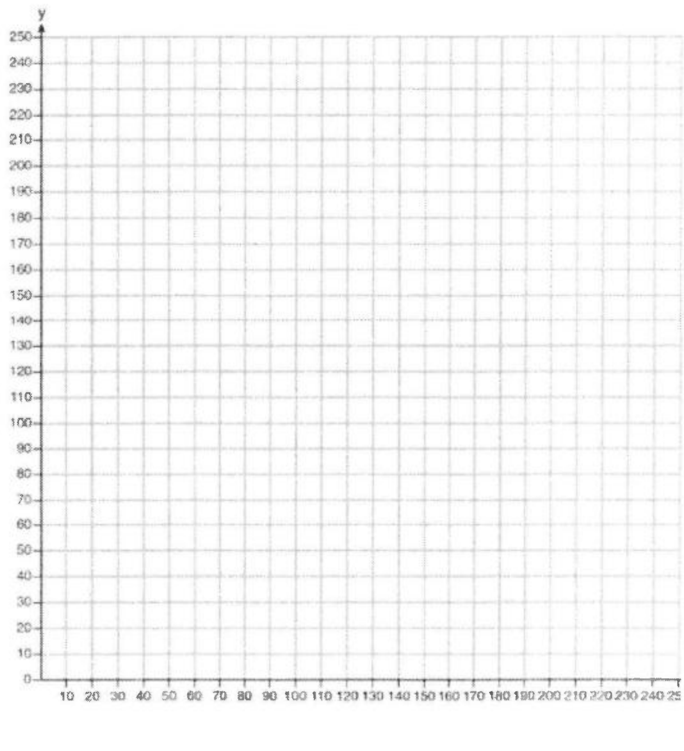

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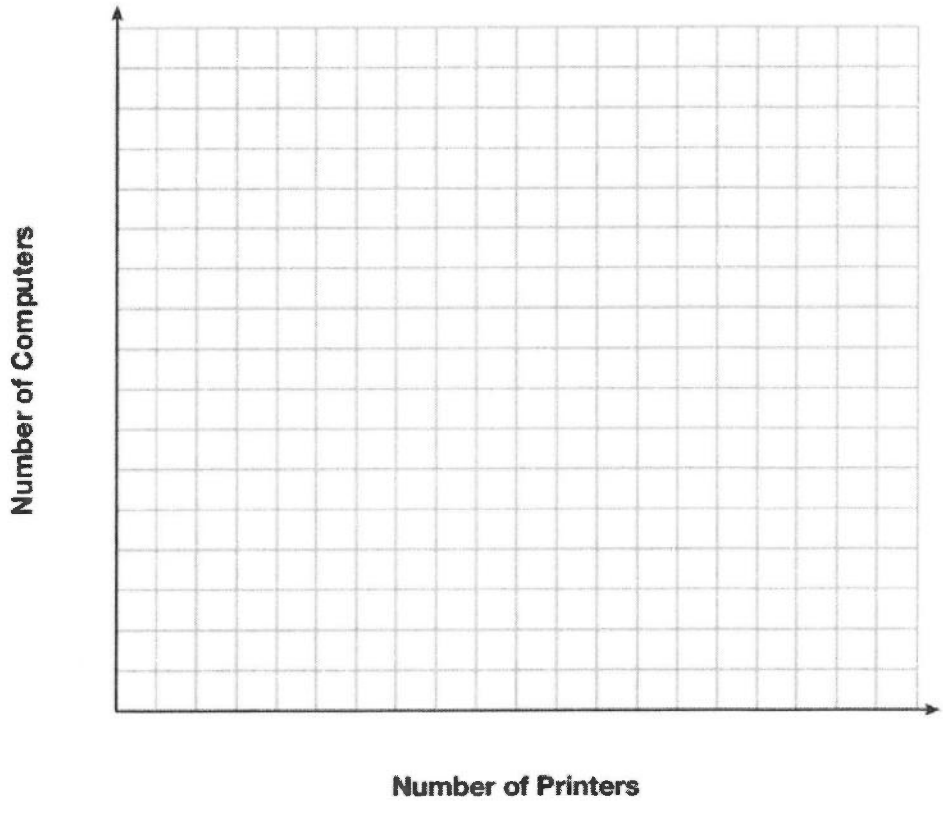

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OBJECTIVE: SWBAT determine and factor out the greatest common factor from a polynomial.

Name: ______________________________ Date: ____________

Lesson 24: Factoring

Vocabulary:

When two numbers are multiplied the answer is called a **product**.
The numbers that were multiplied are called the **factors**.

Numeric Example:

$6 \times 8 = 48$

factor factor product

Algebraic Example:

$\underbrace{(x+1)}_{\text{factor}}\underbrace{(x+2)}_{\text{factor}} = \underbrace{x^2+3x+2}_{\text{product}}$

List all the factors of 18 and 24 in order from least to greatest

	Factors
18	
24	

What is the *largest* factor they have in common? __________

The **greatest common factor, GCF** is the largest number or expression that divides *exactly* into two or more numbers or expressions.

How To Find the GCF:

1) Find all the **factors** of each number.
2) Circle the **common** factors.
3) Choose the **greatest** of those.

Example:

$12x^3$: 1, 2, 3, 4, 6, 12, x, x, x

$16x$: 1, 2, 4, 8, 16, x

GCF: $4x$

OBJECTIVE: SWBAT determine and factor out the greatest common factor from a polynomial.

Practice:

Determine the GCF of each pair of terms below, by listing the factors and verifying in the calculator.

1) $30y^3$ and $20y^2$

2) $35n^2m$ and $21m^2n$

The GCF of a variable is the lowest degree.

In the Calculator:

1) MATH
2) NUM
3) Choice 9 (gcd)
4) ENTER
5) gcd(#, #)

*NO NEGATIVES in the gcd(#, #)

*Three or more numbers: Pick the SMALLEST 2 numbers. Then check to make sure this is a factor of the 3rd number.

Factoring Using the GCF:

*The **first step** in **every** factoring problem is the determine and factor out the GCF*

Solution	Steps
Factor: $2x^2 + 16x$	**1.** Determine the GCF of the polynomial.
GCF: $2x$	**2.** Divide each term in the polynomial by the GCF.
$2x(x + 8)$	**3.** Write as the product of two factors.

Practice:

1) $12x^3 - 18x$

2) $-21x^2y^5 + 14xy^7$

3) $9k^4 + 12k^3 - 6k$

4) $20x^3 - 12x^2 + 28x$

OBJECTIVE: SWBAT determine and factor out the greatest common factor from a polynomial.

Rule for Multiplying Conjugate Pairs: $(a + b)(a - b) = a^2 - b^2$

Perfect Squares

A term is a perfect square when the numerical coefficient is a perfect square and the exponent of each of the variables are even numbers.

Circle the perfect squares:

$49x^2$	$25y^6$	$9x^3$	$20y^4$	$64m^{12}$
$2x^2$	$4x^7$	100	50	x^{10}
b^{12}	x^{21}	$36a^3$	$16x$	9
81	8	$14x^4$	$64x^3$	49

Perfect Squares

$1 = 1 \times 1$
$4 = 2 \times 2$
$9 = 3 \times 3$
$16 = 4 \times 4$
$25 = 5 \times 5$
$36 = 6 \times 6$
$49 = 7 \times 7$
$64 = 8 \times 8$
$81 = 9 \times 9$
$100 = 10 \times 10$
$121 = 11 \times 11$
$144 = 12 \times 12$
$169 = 13 \times 13$
$196 = 14 \times 14$
$225 = 15 \times 15$

Factoring Checklist to use DOTS (Difference of Two Squares):

- 2 terms
- Both terms are perfect squares
- Minus sign

For each of the following, answer yes or no if the expression can be factored using DOTS.

a) $x^2 + 25$	b) $4x^2 - 64y^2$
c) $4x^2 - 25y^2 - 36z^2$	d) $36 - x^2$

OBJECTIVE: SWBAT determine and factor out the greatest common factor from a polynomial.

Difference of Two Perfect Squares:

Factoring is the reverse of multiplying polynomials. When it comes to factoring conjugate pairs, we call it the difference of two perfect squares.

Difference of Two Squares: $a^2 - b^2 = (a + b)(a - b)$

Solution	Steps
Factor: $x^2 - 64$ $()()$ $(x\ )(x\ )$ $(x+\ )(x-\ )$ $(x+8)(x-8)$	**1.** Check for: • Two terms • Both terms are perfect squares • Minus sign **2.** $x^2 - y^2$ is factored as $(x+y)(x-y)$ • **For the square root of numbers:** Press 2^{nd} x^2 in the calculator to find the square root of numbers • **For the square root of variables:** Divide the exponents by 2.

Practice:

Factor the following using DOTS.

a. $t^2 - 36$

a. $b^2 - 100$

b. $121h^4 - 1$

b. $64x^2 - 49y^2$

OBJECTIVE: SWBAT determine and factor out the greatest common factor from a polynomial.

Given a quadratic in standard form $ax^2 + bx + c = 0$, or a similar structured trinomial, we can use ABC factoring when the leading coefficient is equal to 1.

Factoring Checklist to use ABC Factoring

- 3 terms
- Leading coefficient is equal to 1
- Given $ax^2 + bx + c$, two numbers can **add** to b and **multiply** to c

Solution	Steps
Factor: $x^2 + 8x + 15$ $(x + 3)(x + 5)$	**1.** Make sure the trinomial is in standard form (exponents go from greatest to least) **2.** Make sure the polynomial has: • 3 terms • Leading coefficient equal to 1 **3.** List the factors of c **4.** Find the set of factors that add to b.

How to Factor a Trinomial with a Leading Coefficient of 1:

1) Create two sets of parentheses **( ) ( )**.
2) Write the variable at the beginning of each set of parentheses.
3) Determine the factors of the third term, c. →
4) Add each pair of factors together.
5) Choose the factors that add to the coefficient of bx.
6) Write each factor in a different set of parentheses.

Step 3 in the Calculator

1. Press "Y ="
2. Enter $\frac{c}{x}$
3. Go to table.
4. Write coordinate

Factoring Trinomials in the Calculator:

1) Press the "Y =" button.
2) Enter the given trinomial.
3) Go to the table. (Press "2nd" and "Graph.")
4) Find when "Y_1" equals 0.
5) List the corresponding "X"-values.
6) Write the inverse of each x-value in one set of parentheses.

x	y

OBJECTIVE: SWBAT determine and factor out the greatest common factor from a polynomial.

Practice:

Factor the following quadratic expressions.

a) $x^2 + 10x + 16$	b) $x^2 - 5x + 6$
c) $x^2 - 3x - 18$	d) $x^2 + 4x - 1$

Factoring Completely:

1) Determine and factor out the GCF.
2) Examine what is left over: binomial or trinomial.
    a. If binomial, factor as difference of two squares (refer to Lesson 50).
    b. If trinomial, factor into product of binomial (refer to Lesson 51).
3) Write the answer as the product of all factors found.

$5x^2 - 45$	$5x^2 - 35x + 50$
1) GCF: 5	1) GCF: 5
2) Binomial or Trinomial $x^2 - 9$	2) Binomial or Trinomial $x^2 - 7x + 10$
Factor: $(x + 3)(x - 3)$	Factor: $(x - 5)(x - 2)$
3) Final Answer: $5(x + 3)(x - 3)$	3) Final Answer $5(x - 5)(x - 2)$

OBJECTIVE: SWBAT determine and factor out the greatest common factor from a polynomial.

Practice:

Factor each of the following completely. Using your factoring flowchart to help you!

a) $3x^3 - 48x$	b) $x^3 - 3x^2 - 28x$
c) $p^4 - 81$	d) $16x^4 - 64$
e) $4x^2 - 4x - 120$	f) $x^3 - 13x^2 - 30x$

Standards: A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor quadratic expression to reveal the zeros of the function it defines. Standards: A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.

OBJECTIVE: SWBAT identify the features of a quadratic function.

Name: ______________________ Date: ____________

Lesson 25: Graphing Quadratic Functions

A quadratic function is any function that can be placed in the form

$$y = ax^2 + bx + c, \text{ where } a \neq 0$$

The graphs of quadratic functions, $f(x) = ax^2 + bx + c$, are called parabolas.

Parabolas have a shape that resembles, but is *not* the same as, the letter **U**.

Vocabulary:

Parabola: U shape of every quadratic function

Axis of Symmetry: the vertical line given by the graph of the equation, $x = -\frac{b}{2a}$

Vertex: the point where the graph of a quadratic function and its axis of symmetry intersect; also called the turning point

Roots: where the parabola intersects the x-axis; also called zeros and solutions

End Behavior: determines if a parabola opens upward or downward

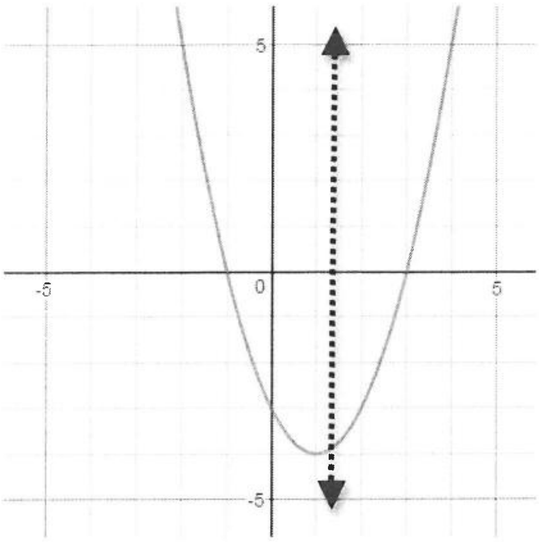
Minimum: the lowest point of an upward parabola; vertex at the bottom

Maximum: the highest point of a downward parabola; vertex at the top

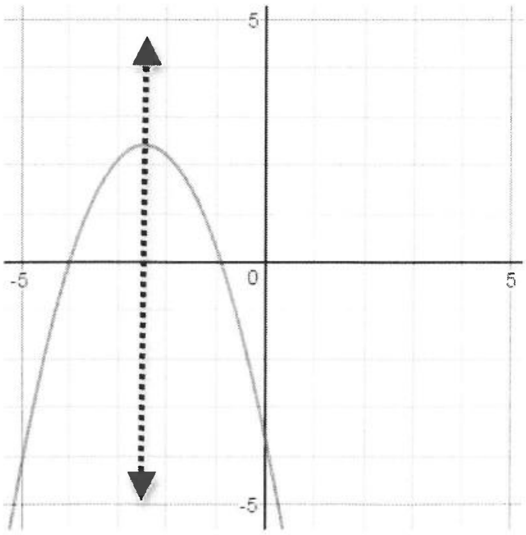
OBJECTIVE: SWBAT identify the features of a quadratic function.

Parabola Features:

$f(x) = ax^2 + bx + c$



$f(x) = -ax^2 + bx + c$

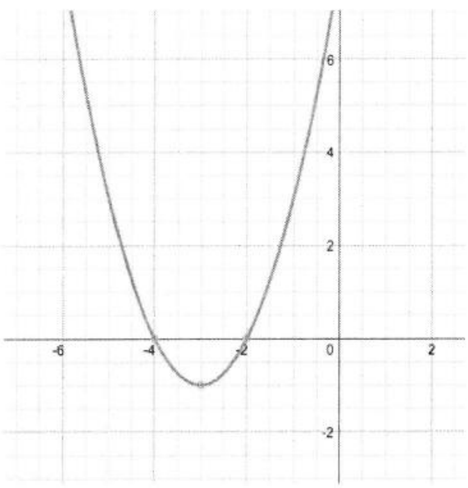


The axis of symmetry and the vertex share an **x** in their name and they share the x-value of a coordinate point.

Vertex:

The vertex of a quadratic function is the point where the axis of symmetry and the function intersect. This is always the maximum or minimum of our graph.

1) Examine the graph of the following quadratic: $f(x) = x^2 + 6x + 8$ and identify the following:



Vertex:

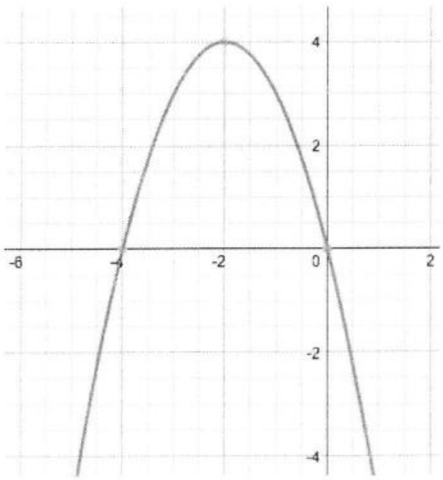
Is it a maximum or a minimum?

Axis of Symmetry:

Roots:

OBJECTIVE: SWBAT identify the features of a quadratic function.

2) Examine the graph of the following quadratic: $f(x) = -x^2 - 4x$ and identify the following:



Vertex:

Is it a maximum or a minimum?

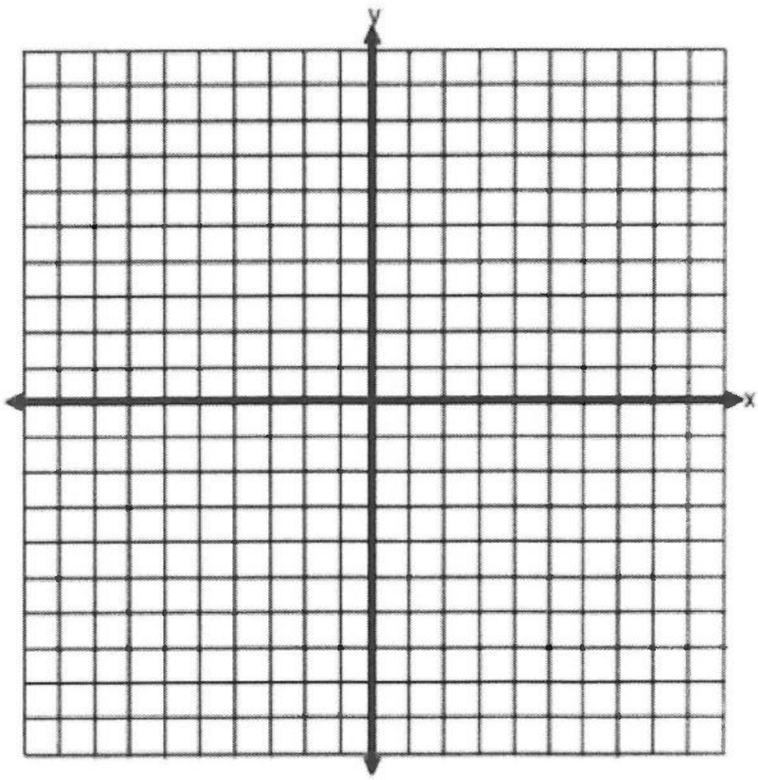
Axis of Symmetry:

Roots:

End Behavior:

End behavior determines if a parabola opens upward or downward.

Example: Graph the equation $y = x^2 - 2x + 1$ on the set of axes below.



Vertex:

Is it a maximum or a minimum?

Axis of Symmetry:

Roots:

Does this parabola open upward or downward?

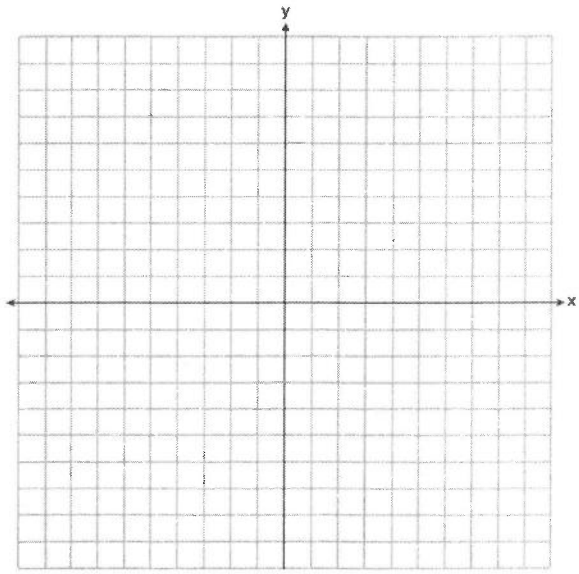
OBJECTIVE: SWBAT identify the features of a quadratic function.

How to Graph a Quadratic Function:

1) Type function into the "Y =" feature.
2) Determine if there is a maximum or minimum.
3) In the table, find the pattern.
4) Copy the table (at least 5 coordinate pairs).
5) Plot the points.
6) Connect the points with a smooth curve, label, and draw arrows.

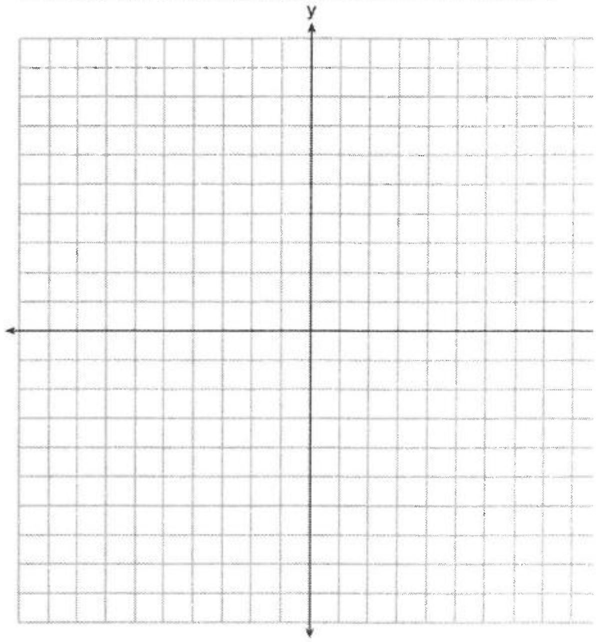
Practice: Graph the following function on the coordinate grid:

$y = x^2 - 6x + 5$



1) Graph the following quadratic function on the set of axes provided. Label and state all of the key features of the graph.

$y = x^2 - 4$



a. Upward or Downward

b. Vertex:

c. Maximum or Minimum

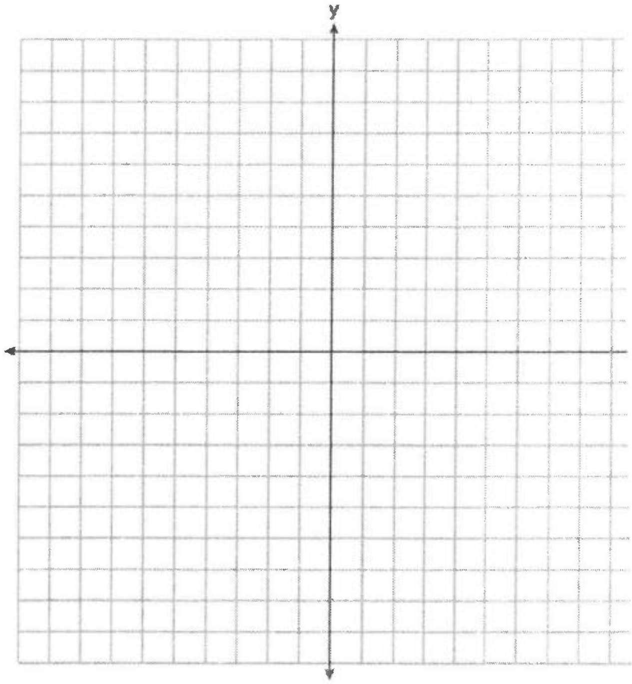
d. Axis of Symmetry:

How many root(s) are there? ____________

What are the root(s)? ____________________

OBJECTIVE: SWBAT identify the features of a quadratic function.

2) Graph the following quadratic function on the set of axes provided. Label and state all of the key features of the graph.



$y = -2x^2 + 4x$

e. Upward or Downward

f. Vertex:

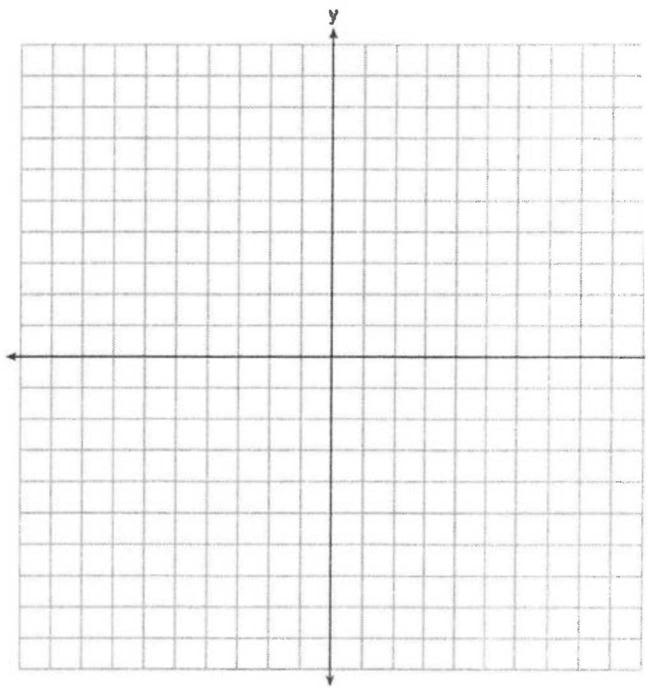
g. Maximum or Minimum

h. Axis of Symmetry:

How many root(s) are there? ____________

What are the root(s)? __________________

3) Graph the following quadratic function on the set of axes provided. Label and state all of the key features of the graph.



$y = \frac{1}{2}x^2 - 4x + 6$

a. Upward or Downward

b. Vertex:

c. Maximum or Minimum

d. Axis of Symmetry:

How many root(s) are there? ____________

What are the root(s)? __________________

OBJECTIVE: SWBAT identify the features of a quadratic function.

Recall:

Our **domain** is all x values.

Our **range** is all y values.

We can use inequality notation or interval notation to represent the domain and range of a graph.

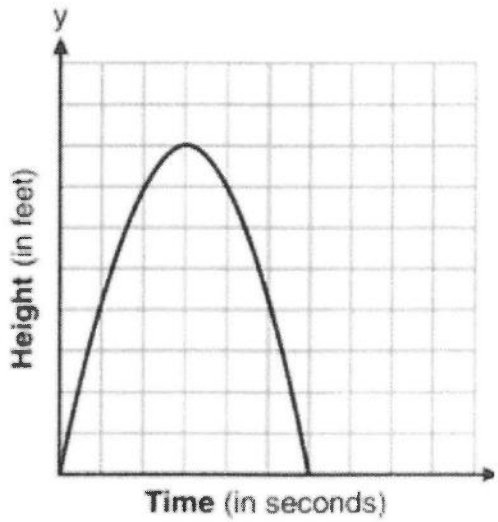
When we see **arrows** on a graph, we use ∞ which always gets (   ).

When we see **open circles** at the end of a graph, we use (   ) or $<$ $>$

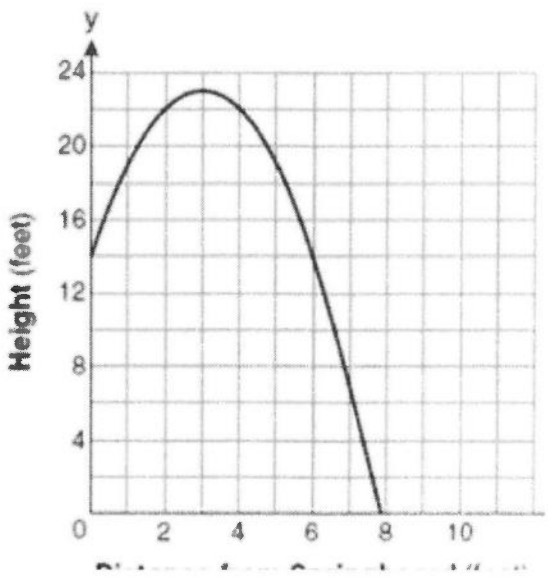
When we see **closed circles** at the end of a graph, we use [   ] or $\leq$ $\geq$

Practice:

1) The graph below represents the parabolic path of a ball kicked by a young child. State the domain and range of the graph.

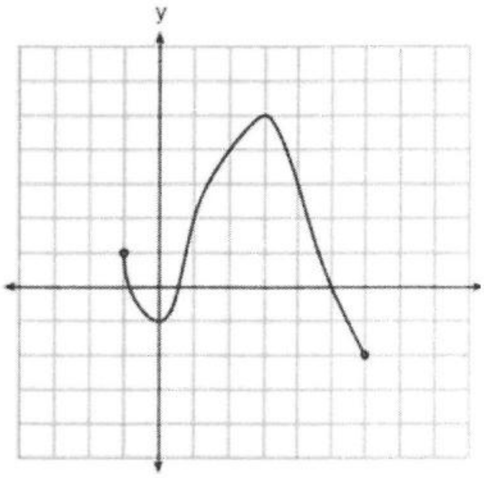


2) A swim team member performs a dive from a 14-foot-high springboard. The parabola below shows the path of her dive. State the domain and range of the graph.



OBJECTIVE: SWBAT identify the features of a quadratic function.

3) What is the range of the function shown below?

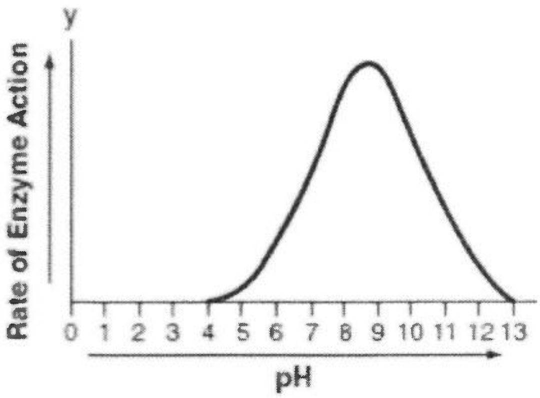


(1) $[-1,5]$ (3) $[-1,6]$

(2) $[-2,1]$ (4) $[-2,5]$

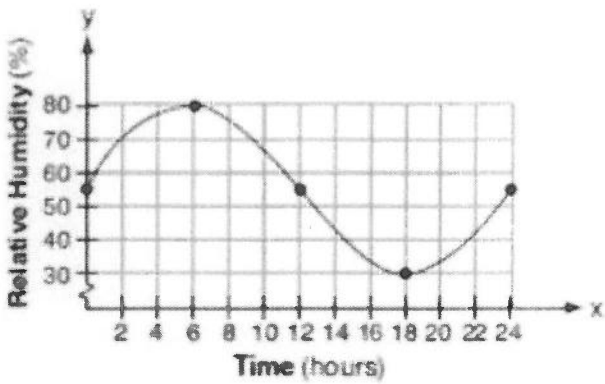
4) The effect of pH on the action of a certain enzyme is shown on the accompanying graph.

State the domain of the graph.



5) A meteorologist drew the accompanying graph to show the changes in relative humidity during a 24-hour period in New York City.

Which of the following represents the domain of the graph?



(1) $0 < x < 24$

(2) $30 < y < 80$

(3) $30 \leq y \leq 80$

(4) $0 \leq x \leq 24$

OBJECTIVE: SWBAT identify the features of a quadratic function.

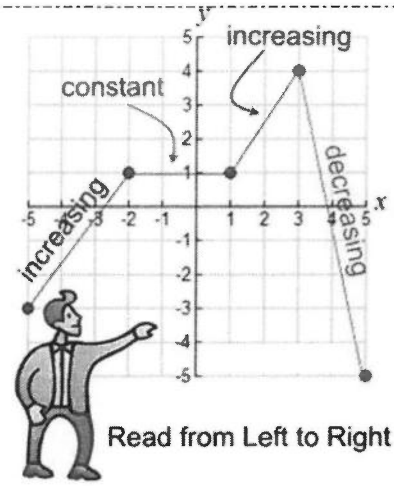
Increasing and Decreasing Intervals

We always read intervals from the x-axis – we read from left to right.

DO NOT read numbers off the y-axis for intervals.

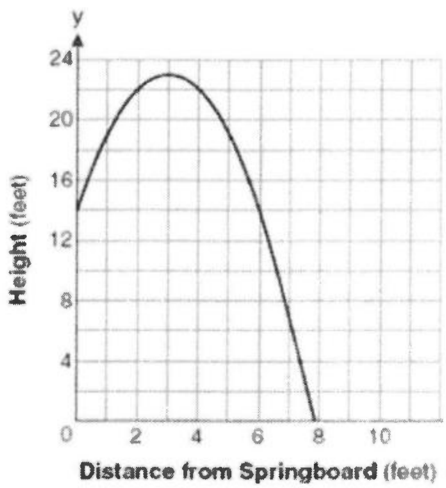
A function is ***increasing***, if as x increases (reading from left to right), y also increases

A function is ***decreasing***, if as x increases (reading from left to right), y decreases



Practice:

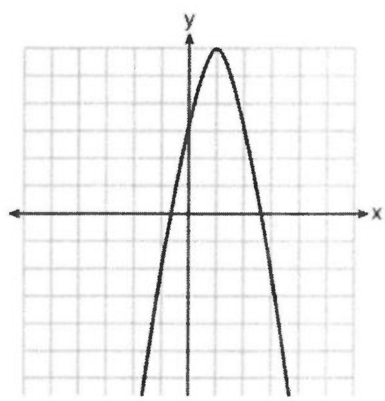
1) A swim team member performs a dive from a 14-foot-high springboard. The parabola below shows the path of her dive.



Write the interval for which the diver's height is increasing.

Write the interval for which the diver's height is decreasing.

2) Gabe graphed the function $f(x)$ as shown below.



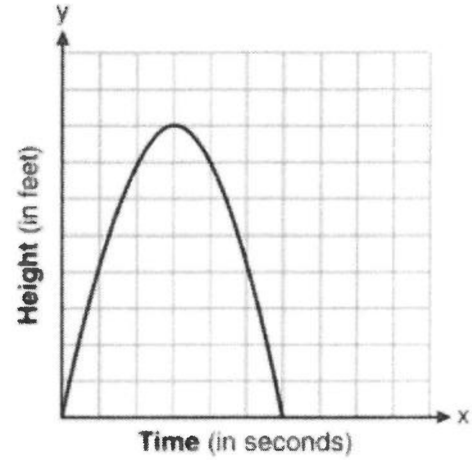
State the interval in which $f(x)$ is *increasing*.

OBJECTIVE: SWBAT identify the features of a quadratic function.

3) The graph below represents the parabolic path of a ball kicked by a young child.

Write the interval for which the ball's height is *increasing*.

Write the interval for which the ball's height is *decreasing*.



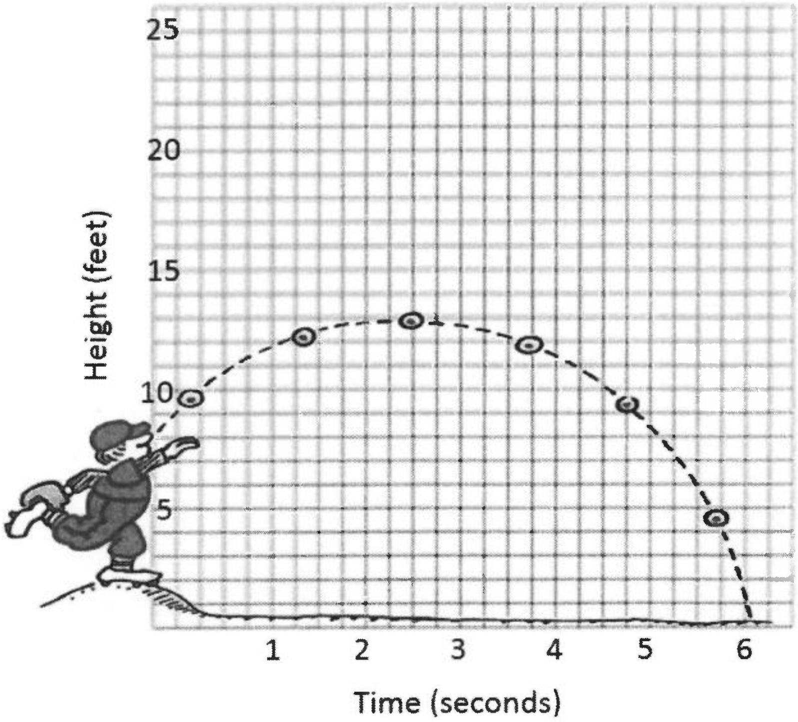
4) A baseball pitcher is throwing the ball to third base in attempt to get a player on the other team to be out. The path of the baseball is shown below.

a) From what height is the baseball thrown?

b) At what time does the baseball reach its maximum height?

c) What is the maximum height of the baseball?

d) After how many seconds does the baseball reach the ground?

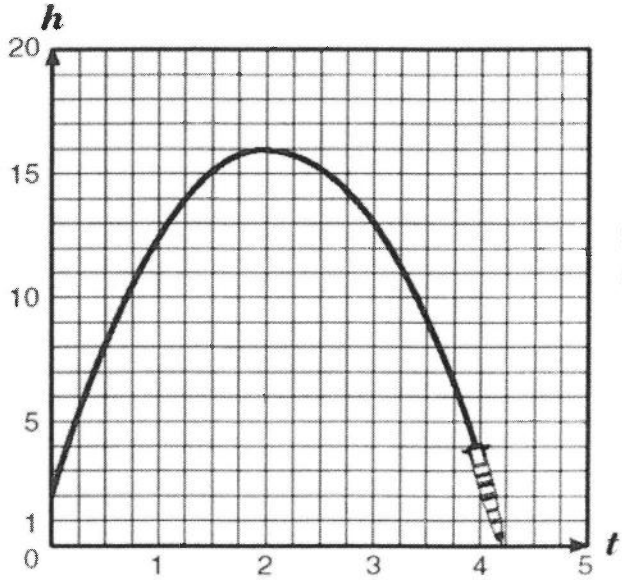


e) How much distance does the baseball travel between 1 second and 4.25 seconds?

f) Over what interval is the baseball always *decreasing*?

OBJECTIVE: SWBAT identify the features of a quadratic function.

3) The graph below shows h, the height in meters of a model rocket, versus t, the time in seconds after the rocket is launched.



a) State the height, in meters, the rocket was initially launched at.

b) After how many seconds does the rocket reach its maximum height?

c) What is the maximum height, in meters, of the rocket?

d) After how many seconds does the rocket reach the ground?

e) State the interval for which the height of the rocket is *increasing*.

f) After how many seconds is the rocket at a height of 14 meters?

g) For how many seconds is the rocket at or above 14 meters?

Standards: CC.A.REI.B.4: Solve quadratic equations in one variable. CC.A.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y=–3x and the circle $x^2+y^2=3$. CC.A.REI.11: Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. CC.A.G.8: Solving Quadratics by Graphing: Find the roots of a parabolic function graphically Note: Only quadratic equations with integral solutions. Focus Standard: CC.A.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y=–3x and the circle x2+y2=3. CC.A.REI.11: Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Standards: HSA.REI.B.4: Solve quadratic equations in one variable. Standards: F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. F.IF.7.a: Graph linear and quadratic functions and show intercepts, maxima, and minima.

OBJECTIVE: SWBAT determine the solution set of a quadratic equation by graphing.

Name: ______________________________ Date: ______________

Lesson 26: Solving Quadratic Functions (Part A)

Solving Quadratics by Graphing:

A quadratic equation is an equation that can be written in the form $y = ax^2 + bx + c$ where $a \neq 0$.

This form is called the **standard form of a quadratic equation.**

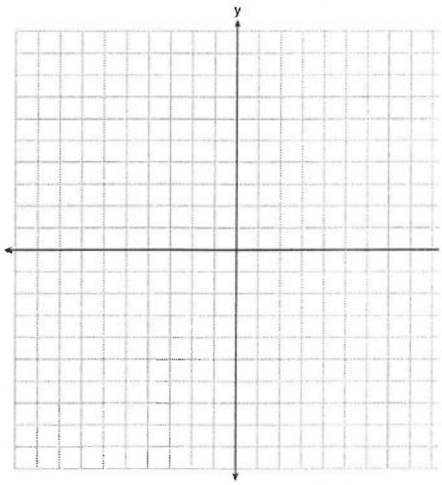
We can solve quadratic equations by graphing and factoring.

To solve a quadric equation by graphing, we must graph the related quadratic function $y = ax^2 + bx + c$. The solutions of a quadratic equation are the x-intercept(s).

1

Guided Practice:

To solve $x^2 - 4 = 0$, we must graph $y = x^2 - 4$.



Where do the x –intercepts occur? ____________________

How many solution(s) are there? _______________

The solutions of $x^2 - 4 = 0$ are the x-intercepts ______ and _____.

******3 CASES******

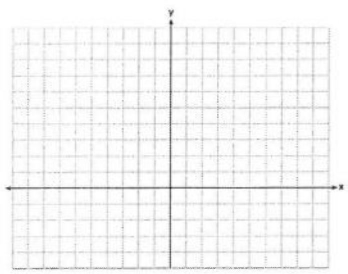
A quadratic equation can have: two, one, OR no real-number solutions.

The solutions are often called zeros of the equation or roots of the function.

What are the solutions of each equation? Use a graph of the related function.

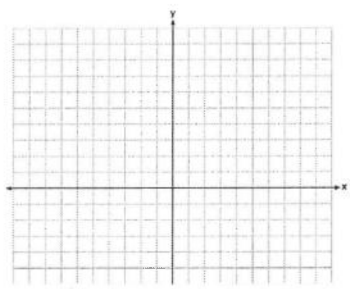
1. $x^2 + 3x + 2 = 0$

Graph: $y =$ _____________



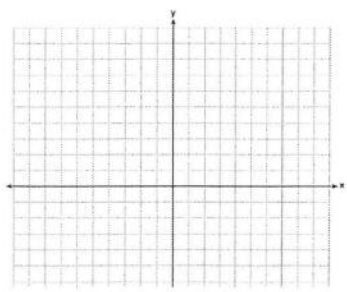
2. $x^2 = 0$

Graph: $y =$ _____________



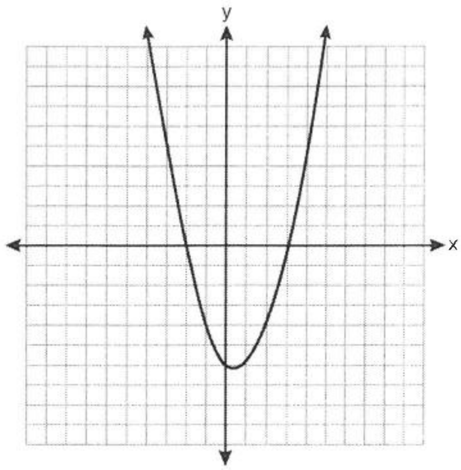
3. $x^2 + 1 = 0$

Graph: $y =$ _____________



OBJECTIVE: SWBAT determine the solution set of a quadratic equation by graphing.

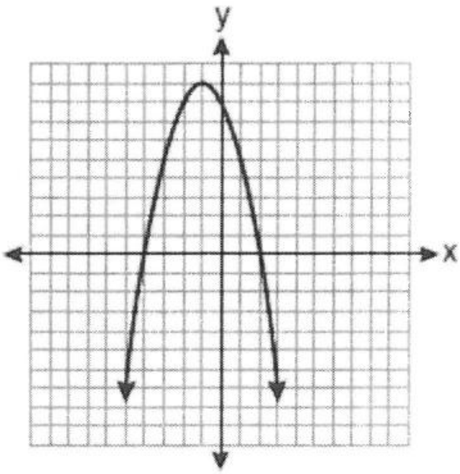
1. A student correctly graphed the parabola shown below to solve a given quadratic equation. What are the root(s) of the quadratic equation associated with this graph?



Root(s): ______________________________

3. The equation $y = -x^2 - 2x + 8$ is graphed on the set of axes below. Based on this graph, what are the roots of the equation $-x^2 - 2x + 8 = 0$?

Root(s): ______________________________



OBJECTIVE: SWBAT determine the solution set of a quadratic equation by graphing.

Solving Quadratic Functions by Factoring:

1) Move all terms to one side of the equal sign.
2) Set the equation equal to 0.
3) Factor the given quadratic.
   a. Look for a GCF.
   b. If the polynomial is a binomial, then use DOTS.
   c. If the polynomial is a trinomial, then use ABC.
4) Set each factor (the polynomial inside each parenthesis) equal to 0.
5) Solve for the variable, x.

Practice:

Example: $x^2 + 4x = 12$

$X^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$X+6 = 0$ $\quad$ $x-2 = 0$

$X = -6$ $\quad$ $x = 2$

Example: $2x^2 - 18 = y$

OBJECTIVE: SWBAT determine the solution set of a quadratic equation by graphing.

Practice:

1) What are the roots of $f(x) = x^2 - 4$?

2) What are the zeros of the function $f(x) = x^2 - 5x - 6$?

3) What is the solution set of the equation $(x - 2)(x + 3) = 0$?

4) Use factoring to determine the zeros of $r(x) = x^2 - 3x - 10$.

5) Use factoring to determine the roots of $f(x) = 4x^2 - 100$

OBJECTIVE: SWBAT determine the solution set of a quadratic equation by graphing.

A perfect square binomial is a binomial factor raised to the power of 2. When we expand and multiply a perfect square binomial, our answer is called a perfect square trinomial.

Perfect square binomial → $(x-2)^2$

$(x-2)(x+2)$

Perfect square $x^2+2x+2x+4$

Perfect square trinomial → x^2+4x+4

Creating a perfect square trinomial is a method called **completing the square.**

How to Complete the Square:

1) Isolate the constant c using inverse operations.
2) Evaluate: $\left(\frac{b}{2}\right)^2$
3) Add the value from step 2 to both sides of the equation.
4) Factor the quadratic as the square of a binomial.
5) Solve for x.

WARNING!!
a must equal 1

So we have

$y=x^2+6x-2$

$$y=x^2+6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2-2$$
$$y=\left(x^2+6x+9\right)-9-2$$
$$y=(x+3)^2-11 \Rightarrow \text{Vertex: } (-3,-11)$$

Abridged Version:

$$\left(x-\frac{b}{2}\right)^2=-c-\left(\frac{b}{2}\right)^2$$

OBJECTIVE: SWBAT determine the solution set of a quadratic equation by graphing.

Vertex Form:

The vertex form of the quadratic function is $y = a(x-h)^2 + k$, where (h, k) is the vertex of the quadratic function and a is the leading coefficient of the quadratic function.

Practice:

1) A student was given the equation $x^2 + 6x - 13 = 0$ to solve by completing the square. The first step that was written is shown below:

$$x^2 + 6x = 13$$

The next step in the student's process was $x^2 + 6x + c = 13 + c$.

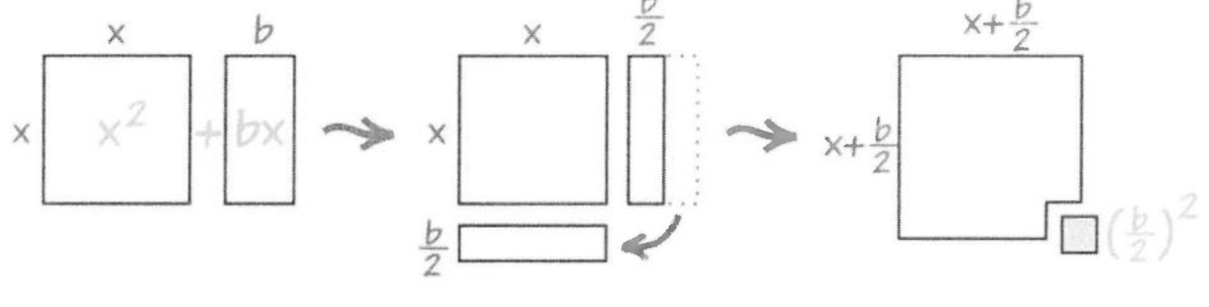
State the value of c that creates a perfect square trinomial. Explain how the value of c is determined.

OBJECTIVE: SWBAT determine the solution set of a quadratic equation by graphing.

2) a) Given the function $f(x) = -x^2 - 8x + 9$, state whether the vertex represents a maximum or minimum point for the function. Explain your answer.

b) Rewrite $f(x)$ in vertex form by completing the square.

Visual Representation:



Standards: A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. b. Complete the square in a quadratic expression to reveal the max and min value of the function it defines. A-REI.B.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x-p)2 = q that has the same solutions. Derive the quadratic formula from this form. **Standards:** A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor quadratic expression to reveal the zeros of the function it defines. A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available. A-REI.B.4 Solve quadratic equations in one variable.

OBJECTIVE: SWBAT determine the solution set of a quadratic function.

Name: ______________________ Date: ____________

Lesson 27: Solving Quadratic Functions (Part B)

Solving Quadratic Functions Using the Square Root Method:

1) Isolate the ax^2 term to the left side of the equal sign.
2) Square root both sides.
3) Use $\pm$ after square rooting the number on the right.
4) Express the roots.

Guided Practice:

Example:

$$x^2 = 81$$

$$\sqrt{81} = \pm 9$$

$$x = \pm 9$$

Practice:

1) Solve for x using square root method: $x^2 - 16 = 0$

2) What is the solution set to the equation $3x^2 = 48$?

3) Find the zeros of $f(x) = x^2 - 64$ using square root method.

OBJECTIVE: SWBAT determine the solution set of a quadratic function.

Quadratic Formula:

When factoring a quadratic equation is not possible, we can either complete the square if $a = 1$ or use the quadratic formula to solve for the roots (always works)!

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before finding $a, b,$ and c, we must first set the equation equal to 0!

Simplify carefully, watch your signs!

Example:

Solve for x using the quadratic formula:

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + 10x + 16 = 0$$

$a = 1$
$b = 10$
$c = 16$

$$x = \frac{-(10) \pm \sqrt{(10) - 4(1)(16)}}{2(1)} = \frac{-10 \pm \sqrt{36}}{2} = \frac{-10 \pm 6}{2}$$

$$x = \frac{-10 + 6}{2} = -2$$

$$x = \frac{-10 - 6}{2} = -8$$

OBJECTIVE: SWBAT determine the solution set of a quadratic function.

Practice:

A) $x^2 - 2x - 24 = 0$

$a =$
$b =$
$c =$

B) $x^2 + 2x - 8 = 0$

$a =$
$b =$
$c =$

OBJECTIVE: SWBAT determine the solution set of a quadratic function.

Closing Assessment:

Ben is asked to solve the following quadratic equation $6x^2 + 11x - 35 = 0$.
His work is shown below:

$$x = \frac{-(11) \pm \sqrt{(11)^2 - 4(6)(-35)}}{2(6)}$$

$$= \frac{-11 \pm \sqrt{121 + 840}}{12}$$

$$= \frac{-11 \pm \sqrt{961}}{12} = \frac{-11 \pm 31}{12}$$

$$= \frac{-11 - 31}{12}, \frac{-11 + 31}{12}$$

$$= -\frac{42}{12}, \frac{20}{12} = -\frac{7}{2}, \frac{5}{3}$$

Do you agree or disagree with Ben's solution? Why?

____

____

Standards: A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor quadratic expression to reveal the zeros of the function it defines. A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available. A-REI.B.4 Solve quadratic equations in one variable.

OBJECTIVE: SWBAT determine the solution set of a quadratic function algebraically from a word problem.

Name: ______________________ Date: ____________

Lesson 28: Modeling with Quadratic Functions

Example:

A foul ball leave the end of a baseball bat and travels according to the formula $h(t) = 64t - 16t^2$, where h is the height of the ball in feet and t is the time in seconds. How long will it take for the ball to hit the ground?

$$64t - 16t^2 = 0$$
$$-16t^2 + 64t = 0$$

$GCF = -16t$

$$-16t(t-4) = 0$$
$$-16t = 0 \qquad t - 4 = 0$$
$$t = 0 \qquad t = 4$$

It will take the ball 4 seconds to hit the ground.

******HINT: Use the Quadratic Formula for the rest of the problems in this lesson.******

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Practice:

1) A rocket is shot vertically into the air. Its height, h, at any time, t, in seconds can be modeled by the equation $h = -16t^2 + 184t$. Determine algebraically, the number of seconds it will take for the rocket to reach a height of 529 feet.

OBJECTIVE: SWBAT determine the solution set of a quadratic function algebraically from a word problem.

2) A cliff diver on a Caribbean island jumps from a height of 105 feet, with an initial upward velocity of 5 feet per second. An equation that models the height, $h(t)$, above the water in feet of the diver in time elapsed, t, in seconds is $h(t) = -16t^2 + 5t + 105$. How many seconds, to the *nearest hundredth*, does it take the diver to fall 45 feet below his starting point?

3) During an enthusiastic game of hacky sack, Eli kicks the hacky sack straight up into the air and the path of the hacky sack can be modeled by the equation $h = -16t^2 + 100t + 3$ where h is the height in feet and t is the time in seconds. How long will it take for the hacky sack to return to his foot 3 feet above the ground?

OBJECTIVE: SWBAT determine the solution set of a quadratic function algebraically from a word problem.

Closing Assessment:

Carmine drops a ball at shoulder height from the top of a builder. If the ball can be modeled by the equation $h = -16t^2 + 40$, where h is the height in feet and t is the time in seconds, how long will it take, to the *nearest tenth of a second*, to hit the ground.

Standards: CC.A.REI.B.4: Solve quadratic equations in one variable. CC.A.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y=–3x and the circle $x^2+y^2=3$. CC.A.REI.11: Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. CC.A.G.8: Solving Quadratics by Graphing: Find the roots of a parabolic function graphically Note: Only quadratic equations with integral solutions.

OBJECTIVE: SWBAT analyze exponential functions and graphs.

Name: ______________________________ Date: ________________

Lesson 29: Exponential Functions

Exponential Functions

An equation is exponential when the exponent is a variable!

An **exponential function** with base b is defined by:

$$f(x) = ba^x.$$

b represents our initial value.

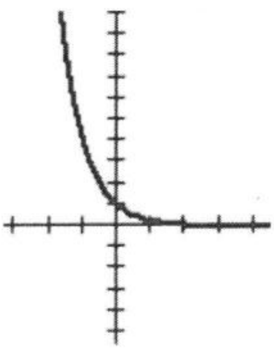
a represents what we multiply by.

When $a > 1$, the graph increases. This is known as exponential growth.

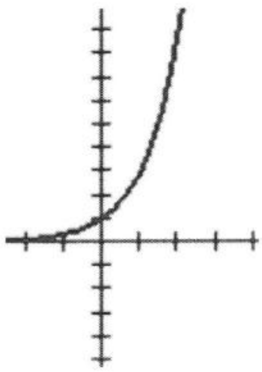
When $0 < a < 1$, the graph decreases. This is known as exponential decay.

Ex 1) Given the exponential functions graphed below, state whether each represents exponential growth or decay.

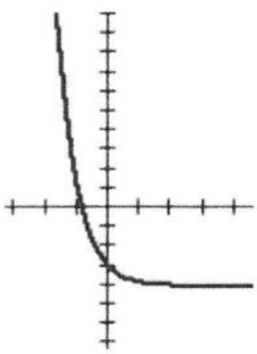
A) ________________



B) ________________



C) ________________



Ex 2) Given the exponential functions shown below, state the initial value and circle whether it will be growth or decay.

A) $y = 2(3)^x$

Initial value: ________

Growth or Decay

B) $y = 4\left(\frac{1}{3}\right)^x$

Initial value: ________

Growth or Decay

C) $y = \frac{1}{2}(2)^x$

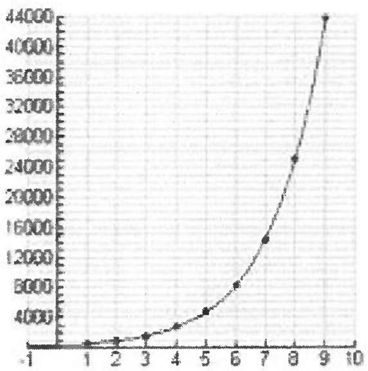
Initial value: ________

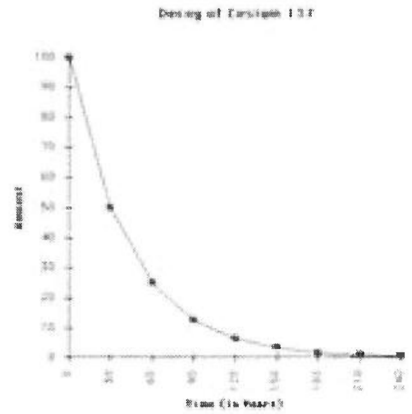
Growth or Decay

OBJECTIVE: SWBAT analyze exponential functions and graphs.

Growth vs. Decay:

There are 2 types of exponential functions, growth and decay. A graph of exponential growth will increase, while a graph of exponential decay will decrease.





A graph that goes up means exponential growth.

A graph that goes down means exponential decay.

USE FORMULA: $y = b(1 + r)^x$

*Notice plus sign for growth.

Growth: $(1 + r) > 1$

USE FORMULA: $y = b(1 - r)^x$

*Notice minus sign for decay.

Decay: $0 < (1 - r) < 1$

b = amount (initial amount started with)

r = rate of change (% changed to a decimal)

x = time

OBJECTIVE: SWBAT analyze exponential functions and graphs.

Exponential Growth

Any quantity that grows by a fixed **percent** at regular intervals is called exponential growth! The formula used for exponential growth is:

$$y = b(1 + r)^t$$

y = the final amount

b = the initial amount

r = the percent rate (over 100)

t = the time

Key Words for Growth

- **Grow**
- **Increases**
- **Interest earned**
- **Double**
- **Triple**
- **Appreciation**

Practice:

1) Dylan invested $600 in a savings account at a 1.6% annual interest rate. He made no deposits or withdrawals on the account for 2 years. The interest was compounded annually. Find, to the *nearest cent*, the balance in the account after 2 years.

$$600\left(1 + \frac{1.6}{100}\right)^2 =$$

2) A flu outbreak hits your school on Monday, with an initial number of 20 ill students coming to school the number of ill students increases by 25% per hour. How many students will be ill after 6 hours?

OBJECTIVE: SWBAT analyze exponential functions and graphs.

Finding the Growth Rate:

Use the exponential growth factor $(1+r)$ to solve for r!
Rule: $1 - a * 100 = r$

1) The function $V(t) = 1350(1.017)^t$ represents the value $V(t)$, in dollars, of a comic book t years after its purchase. The yearly **rate** of appreciation is

(1) 17% (3) 1.017%

(2) 1.7% (4) 0.017%

2) The equation $A = 1300(1.02)^7$ is being used to calculate the amount of money in a savings account. What does the 1.02 represent in this equation?

(1) 0.02% decay (3) 2% decay

(2) 0.02% growth (4) 2% growth

Exponential Decay

Any quantity that decays by a fixed **percent** at regular intervals is called exponential decay! The formula used for exponential decay is:

$$y = b(1-r)^t$$

y = the final amount

b = the initial amount

r = the percent rate (over 100)

t = the time

Key Words for Decay

- **Decay**
- **Decreases**
- **Half-life**
- **Depreciation**
- **Decays by a factor of** $\frac{1}{3}, \frac{1}{4}$, **etc.**

OBJECTIVE: SWBAT analyze exponential functions and graphs.

Practice:

1) Cody buys a pair of Air force 1s for $160. The shoes depreciate at a rate of 32.3% each year. If Cody wears the shoes for 2 years, how much could he sell the shoes for? Round your answer to the *nearest cent.*

$$160\left(1-\frac{32.3}{100}\right)^2 =$$

2) Jill bought a pair of Beats by Dre for $200. The headphones depreciate at a rate of 27.5% each year. Find how much Jill could sell the beats for after 4 years? Round your answer to the *nearest dollar.*

3) A vlogger currently has 3,250,000 viewers each day on their channel. Due to them getting the flu their rate of viewers will decrease at a rate of 16% each hour. How many viewers can they expect in 2 days?

OBJECTIVE: SWBAT analyze exponential functions and graphs.

Finding the Decay Rate:

Use the exponential decay factor $(1-r)$ to solve for r!
Rule: $1-a*100=r$

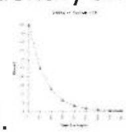
1) Milton has his money invested in a stock portfolio. The value, $v(x)$, of his portfolio can be modeled with the function $v(x)=30{,}000(0.78)^x$, where x is the number of years since he made his investment. Which statement describes the rate of change of the value of his portfolio?

(1) It decreases 78% per year.
(2) It decreases 22% per year.
(3) It increases 78% per year.
(4) It increases 22% per year.

2) The value $v(t)$, of a car depreciates according to the formula $v(t)=P(0.85)^t$, where P is the purchase price of the car and t is the time, in years, since the car was purchased. State the percent that the value of the car *decreases* by each year. Justify your answer.

Closing Assessment:

Identify and circle if the following problems represent exponential **Growth** or **Decay**

1.  **Growth** or **Decay**

2. $y=2(1.29)^x$ **Growth** or **Decay**

3. $y=0.04^x$ **Growth** or **Decay**

4. A cars value depreciated. **Growth** or **Decay**

Standards: CCSS.MATH.CONTENT.HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)ᵗ, y = (0.97)ᵗ, y = (1.01)12ᵗ, y = (1.2)ᵗ/10, and classify them as representing exponential growth or decay.

OBJECTIVE: SWBAT identify and analyze the type of exponential function from a word problem.

Name: ______________________ Date: ______________

Lesson 30: Modeling with Exponential Functions

Exponential Formulas:

GROWTH: $y = b(1 + r)^x$

DECAY: $y = b(1 - r)^x$

$b = initial\ amount$

$r = rate\ \%$

$x = time$

Example: Samantha invested \$700 in a savings account at a 1.75% annual interest rate. She made no deposits or withdrawals on the account for 3 years. The interest was compounded annually. Find, to the *nearest dollar*, the balance in the account after 3 years.

Steps

1) Determine if its growth or decay
2) Write the proper formula
3) Identify a, r (as a decimal), and t
4) Substitute and solve.

$$\mathbf{700\left(1 + \frac{1.75}{100}\right)^3 = 737.3968}$$

$$\mathbf{\$737.00}$$

Practice:

1. The number of carbon atoms in a fossil is given by the function $y = 5100(0.95)^x$, where x represents the number of years since being discovered. What is the percent of change each year? Explain how you arrived at your answer.

OBJECTIVE: SWBAT identify and analyze the type of exponential function from a word problem.

2. A laboratory technician studied the population growth of a colony of bacteria. He recorded the number of bacteria every other day, as shown in the partial table below.

t (time, in days)	0	2	4
f(t) (bacteria)	25	15,625	9,765,625

Which function would accurately model the technician's data?

(a) $f(t) = 25^t$
(b) $f(t) = 25^{t+1}$
(c) $f(t) = 25t$
(d) $f(t) = 25(t+1)$

3. The value in dollars, $v(x)$, of a certain car after x years is represented by the equation $v(x) = 25{,}000(0.86)^x$. To the nearest dollar, how much more is the car worth after 2 years than after 3 years?

4. The amount, A, in grams of a radioactive material that is decaying can be modeled by the equation $A(d) = 450(0.88)^d$, where d is the number of days since it started to decay. Find the difference in weight between day 5 and day 12.

OBJECTIVE: SWBAT identify and analyze the type of exponential function from a word problem.

5. The table below represents the function F.

x	3	4	6	7	8
F(x)	9	17	65	129	257

The equation that represents this function is

(a) $F(x) = 3^x$
(b) $F(x) = 3x$
(c) $F(x) = 2^x + 1$
(d) $F(x) = 2x + 3$

6. The equation $B = 1400(1.05)^5$ is being used to calculate the amount of money in a savings account. What does 1.05 represent in this equation?

(1) 0.05% decay
(2) 0.05% growth
(3) 5% decay
(4) 5% growth

7. Milo invested $850 in an account with a 1.24% annual interest rate. He made no deposits or withdrawals on the account for 5 years. If interest was compounded annually, which equation represents the balance in the account after 5 years?

(1) $A = 850(1 + 1.24)^5$
(2) $A = 850(1 - 1.24)^5$
(3) $A = 850(1 + 0.0124)^5$
(4) $A = 850(1 - 0.0124)^5$

OBJECTIVE: SWBAT identify and analyze the type of exponential function from a word problem.

8. Hank has his money invested in a stock portfolio. The value, $v(x)$, of his portfolio can be modeled with the function $v(x) = 45{,}000(0.83)^x$, where x is the number of years since he made his investment. Which statement described the rate of change of the value of his portfolio?

(1) It decreases by 17% each year.

(2) It decreases by 82% each year.

(3) It increases by 17% each year.

(4) It increases by 82% each year.

Closing Assessment:

The function $V(t) = 1350(1.017)^t$ represents the value $V(t)$, in dollars, of a comic book t years after its purchase. What is the yearly rate of appreciation of the comic book? Explain why.

Standards: CCSS.MATH.CONTENT.HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.

OBJECTIVE: SWBAT identify whether a function is linear or exponential.

Name: ______________________________ Date: ______________

Lesson 31: Linear vs. Exponential Functions

Linear vs. Exponential Equations

Linear	Exponential
$y = mx + b$ x is raised to a power of 1	$y = ab^x$ x is the exponent/power

State whether each of the following function are linear or exponential.

a) $f(x) = 2x - 1$

b) $g(x) = 2(2)^x$

c) $h(x) = 3x$

d) $j(x) = 2^x + 3$

e) $a(x) = 3\left(\frac{1}{2}\right)^x$

f) $b(x) = 1 + x$

Linear vs. Exponential Functions:

Linear

We see equal differences over equal intervals.

In other words, we see a pattern of **addition** or **subtraction** in the y-values.

Example:

x	−3	−2	−1	0	1	2	3
y	14	10	6	2	−2	−6	−10

In the table, we see a pattern of **subtracting 4**

Exponential

We see equal factors over equal intervals.

In other words, we see a pattern of **multiplication** in the y-values.

Example:

x	−3	−2	−1	0	1	2	3
y	4	8	16	32	64	128	256

In the table, we see a pattern of **multiplying 2**

OBJECTIVE: SWBAT identify whether a function is linear or exponential.

Exploring Tables:

(a)

x	-2	-1	0	1	2
y	32	16	8	4	2

Type: ______________________________

Equation: ___________________________

(b)

x	-2	-1	0	1	2
y	32	16	0	-16	-32

Type: ______________________________

Equation: ___________________________

State whether the following tables are linear or exponential.

a)

x	f(x)
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

b)

x	t(x)
-3	10
-1	7.5
1	5
3	2.5
5	0

c)

x	f(x)
1	12
2	19
3	26
4	33

d)

x	f(x)
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

e)

x	f(x)
-1	-3
0	-1
1	1
2	3
3	5

OBJECTIVE: SWBAT identify whether a function is linear or exponential.

Linear vs. Exponential Situations

Linear	Exponential
• Addition • Subtraction • Increase by a constant amount • Decrease by a constant amount • Deducts • Equal differences over equal intervals	• Multiplication • Percent rate • Half-life • Interest • Doubling, tripling, taking half, etc. • Depreciates • Grows/decays by a factor

State whether the following situations are modeled by a linear or exponential function.

a) A sunflower grows at a rate of 3.5 cm per day. ________________

b) The value of a car depreciates at a rate of 15% per year after it is purchased. _____________

c) The amount of bacteria in a culture triples every two days during an experiment. ___________

d) A farmer plants two beds of flowers every day. ________________

e) A bank account balance grows at a rate of 6% each year. ________________

f) The cost of joining a gym that charges an initial fee plus $40 per month. ________________

g) The concentration of medicine in a person's body that decays by a factor of one-third every hour. ________

h) A number of players left in a tennis tournament decreases by one-half of the players in each round. ______

i) A student saves $300 every month in a savings account at a local bank. ________________

OBJECTIVE: SWBAT identify whether a function is linear or exponential.

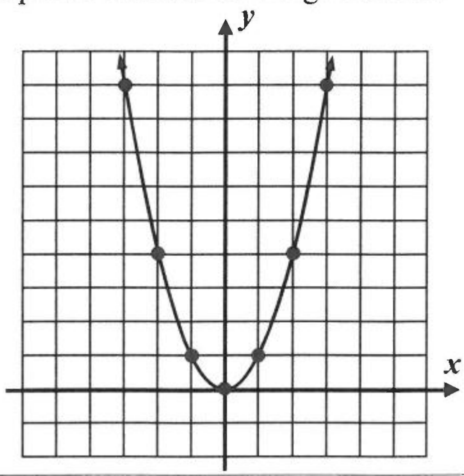
Quadratic Example:

Consider the function $f(x) = x^2$

(a) Fill out the table below without using your calculator.

x	−3	−2	−1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9

(b) Graph the function on the grid shown.



(c) What is the **range** of this quadratic function?

The range is the set of all outputs (y-values). Notice here that no points are below the x-axis.

Range: $y \geq 0$ or $[0, \infty)$

Closing Assessment:

1. For each of the following problems a table of values is given where the change in x is 1. For each, determine if the table represents a linear function, of the form $y = mx + b$, or an exponential function, of the form $y = b(a)^x$. Then, write its equation.

(a)

x	−1	0	1	2	3
y	4	7	10	13	16

(b)

x	0	1	2	3	4
y	2	6	18	54	162

Type: ______________________

Equation: ______________________

Type: ______________________

Equation: ______________________

Standards: F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratic, or (more generally) as a polynomial function. F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

OBJECTIVE: SWBAT find a term in a sequence.

Name: ______________________________ Date: ______________

Lesson 32: Sequences

Sequences – Basic Information

A **sequence** is an ordered list. Each number in a sequence is called a term, an element, or a member.

Terms are referenced in a subscripted form (written below), where the natural number subscripts refer to the location or position of the term in the sequence.

Sequence:	**1,**	**5,**	**9,**	**13,**	**17,**	**21,**	**...**
Notation for terms of the sequence:	a_1	a_2	a_3	a_4	a_5	a_6	

The first term would be a_1. The subscripted 1 means first term.

The second term would be a_2, the third term would be a_3, the nth term is a_n.

Sequences are Functions!

A functional form of a sequence is represented by $f(1), f(2), f(3), \ldots f(n)$.

$f(1)$ is our first term. $f(2)$ is our second term, $f(3)$ is our third term, and $f(n)$ is our nth term.

Guided Practice:

Given the sequence $\{1, 5, 9, \ldots\}$ fill in the table below.

Term Number	Term	Subscript Notation	Function Notation
1			
2			
3			
n			

OBJECTIVE: SWBAT find a term in a sequence.

Arithmetic Sequences

In an **arithmetic sequence** the difference between consecutive terms (terms that are next to each other) is constant. This is called the **common difference** and is represented with the letter d.

Practice:

Ex 1) What is the common difference of the arithmetic sequence $\{-7, -3, 1, 5, \ldots\}$

Ex 2) What is the seventh term of the arithmetic sequence $\{2, 5, 8, 11, \ldots\}$?

Ex 3) State the common difference in each arithmetic sequence. Then, state the next two terms.

a) $\{5, 11, 17, 23, \ldots\}$	**b)** $\{20, 15, 10, 5, \ldots\}$	**c)** $\{-3, -7, -11, -15, \ldots\}$

OBJECTIVE: SWBAT find a term in a sequence.

Recall: A sequence is a function whose domain is a subset of natural numbers (i.e. 1, 2, 3,...). A sequence is shown as an ordered list of numbers, called terms. Sequences are functions so the key is to think of the input as the number's place in line and the output is the actual number in the list.

An **arithmetic sequence** is a sequence in which you **add** or **subtract** the same value to get from one term to the next. We can use the formula to find any term in an arithmetic sequence!

$$a_n = a_1 + (n - 1)d$$

a_n is the nth term

a_1 is the first term

n is the position of the nth term

d is the common difference

Ex 1) What is the 27th term of the arithmetic sequence $\{3, 5, 7, 9, \ldots\}$?

Step 1: List out n, a_1, and d.

$n = 27$
$a_1 = 3$
$d = 2$

Step 2: Substitute the values into arithmetic formula.

$a_n = a_1 + (n - 1)d$
$a_{27} = 3 + (27 - 1)(2)$

Step 3: Solve.

$a_{27} = 55$

The 27th term is 55.

Ex 2) What is the 30th term of the arithmetic sequence $\{-3, 1, 5, \ldots\}$?

OBJECTIVE: SWBAT find a term in a sequence.

Writing an Arithmetic Sequence:

$$a_n = a_1 + (n-1)d$$

a_n is the nth term

a_1 is the first term

n is the position of the nth term

d is the common difference

If we are given a sequence, we can write an equation that can be used to find any term in the sequence! We simply substitute in our first term and common difference.

Ex 3) Write an equation that can be used to find a term in the sequence $-6, -10, -14, -18, \ldots$

Step 1: State a_1 and d

$a_1 = -6$
$d = -4$

Step 2: Substitute into the formula $a_n = a_1 + (n-1)d$

$a_n = -6 + (n-1)(-4)$

Step 3: Simplify and write as a linear equation.

$a_n = -6 - 4n + 4$

$\boldsymbol{a_n = -4n - 2}$

Ex 4) Which of the following could be used to find a term in the sequence $7,\ 9, 11, 13, \ldots$?

(1) $f(n) = 2n + 5$

(2) $f(n) = 2n + 9$

(3) $f(n) = 2n + 7$

(4) $f(n) = 7n + 2$

Ex 5) The first term in an arithmetic sequence is -6 and the common difference is 8. Which is an equation for the nth term of this sequence?

(1) $a_n = 8n + 10$

(2) $a_n = 8n - 14$

(3) $a_n = 16n + 10$

(4) $a_n = 16n - 38$

OBJECTIVE: SWBAT find a term in a sequence.

A **geometric sequence** is a sequence in which you **multiply** the same value to get from one term to the next. We can use the formula to find any term in an arithmetic sequence!

$$a_n = a_1 \cdot r^{n-1}$$

a_n is the nth term

a_1 is the first term

n is the position of the nth term

r is the common ratio

Finding a Common Ratio:

To find the common ratio (what we multiply by) we take any two consecutive terms and divide $\frac{right}{left}$

Ex 1) Find the common ratio of each:

a) $1, 4, 16, 64, \ldots$

b) $81, 27, 9, 3, \ldots$

Ex 2) Given the geometric sequence $-2, -4, -8, \ldots$ find a_7.

Ex 3) What is the ninth term of the geometric sequence $1, 4, 16, \ldots$?

Ex 4) What is the formula for the nth term of the sequence $54, 18, 6, \ldots$

(1) $a_n = 6\left(\frac{1}{3}\right)^n$

(2) $a_n = 6\left(\frac{1}{3}\right)^{n-1}$

(3) $a_n = 54\left(\frac{1}{3}\right)^n$

(4) $a_n = 54\left(\frac{1}{3}\right)^{n-1}$

OBJECTIVE: SWBAT find a term in a sequence.

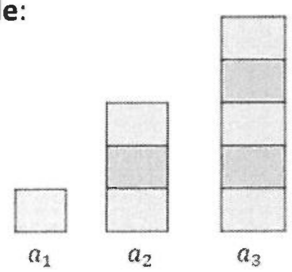
Arithmetic vs. Geometric

Arithmetic Sequences

We see a pattern of addition or subtraction.
Arithmetic sequences are **linear!**

Numerical Example: $3, 5, 7, 9, \ldots$

Picture Example:



Sequence Formula

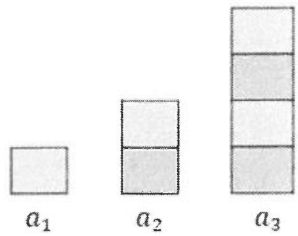
$$\boldsymbol{a_n = a_1 + (n-1)d}$$

Geometric Sequences

We see a pattern of multiplication or division.
Geometric sequences are exponential!

Numerical Example: $2, 6, 18, 54, \ldots$

Picture Example:



Sequence Formula

$$\boldsymbol{a_n = a_1 \cdot r^{n-1}}$$

Ex 1) For each of the following:

- State whether it is arithmetic or geometric
- If arithmetic, state the common difference. If geometric, state the common ratio
- Find a_9

a) $4, 1, -2, -5, \ldots$	b) $\frac{1}{4}, 1, 4, 16, \ldots$
c) $14, 21, 28, 35, \ldots$	d) $100, 50, 25, \ldots$

OBJECTIVE: SWBAT find a term in a sequence.

Certain sequences, not all, can be defined in a recursive form. In a **recursive formula**, each term is defined as a function of its previous term.
A recursive formula designates the starting term, $a_1$and the n^{th} term of the sequence a_n, as an expression containing the previous term, a_{n-1}.

Given Term	Previous Term
a_4	
a_n	
a_{n+1}	
a_{n+4}	
$f(6)$	
$f(n)$	
$f(n+1)$	

A **recursive formula** always has **two** parts:

1. a_1 (the first term)

2. the rule for a_n as a function of a_{n-1} (the previous term)

Ex 1) Consider the recursively defined sequence shown below.

$$a_1 = 15$$
$$a_n = a_{n-1} + 5$$

Our two parts given:

1. Starting term= 15
2. Our rule: A term in the sequence is a_{n-1} (the last term) plus 5.

Find the next five terms of the sequence using this rule.

$$a_1 = 15$$
$$a_2 = a_{2-1} + 5 = a_1 + 5 = 15 + 5 = 20$$
$$a_3 = a_2 + 5 = 20 + 5 = 25$$
$$a_4 = a_3 + 5 = 25 + 5 = 30$$
$$a_5 = 30 + 5 = 35$$
$$a_6 = 35 + 5 = 40$$

OBJECTIVE: SWBAT find a term in a sequence.

Ex 2) Find the next three terms of the recursively defined sequence:

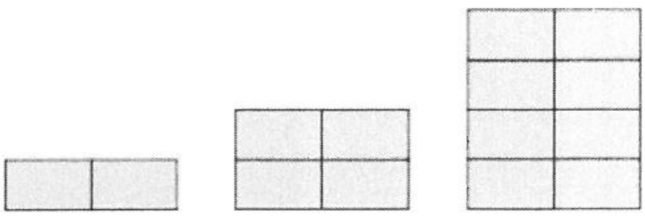
$$a_1 = 29$$
$$a_n = a_{n-1} + 10$$

Ex 3) Find the third term of the recursively defined sequence:

$$a_1 = \frac{1}{2}$$
$$a_n = 8a_{n-1}$$

Closing Assessment:

Is the pattern of rectangles below arithmetic or geometric? If arithmetic state, the common difference. If geometric, state the common ratio.



Standards: F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratic, or (more generally) as a polynomial function. F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. Standards: F-BF.A.1 Write a function that describes a relationship between two quantities. F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

OBJECTIVE: SWBAT identify, describe, and graph special functions.

Name: ______________________________ Date: ______________

Lesson 33: Special Functions

Cubic Functions:

A cubic function is any function that can be placed in the form $y = ax^3 + bx^2 + cx + d, where\ a \neq 0$

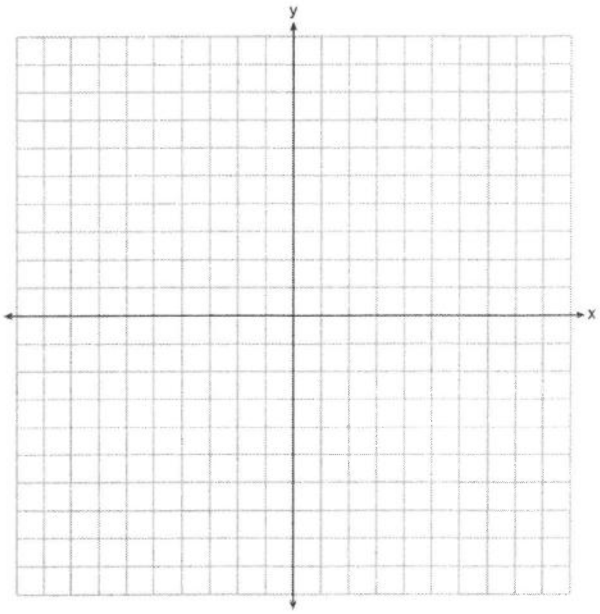
The constant d is the y −intercept of the function.

A cubic function may have 1, 2, or 3 x-intercepts.

How to Graph a Cubic Function:

1) Press "$Y =$"
2) Type in function. For cubic, use carrot key.
3) Create a table of values.
4) Plot and connect the points.
5) Label your graph.

Create the graph of the following function:
$y = x^3$



a) What is the x −intercept(s)?

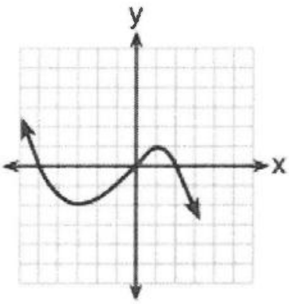
b) What is the y −intercept(s)?

OBJECTIVE: SWBAT identify, describe, and graph special functions.

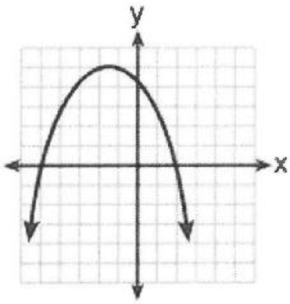
Practice:

1) A polynomial function contains the factors x, $x - 2$, and $x + 5$. Which graph(s) below could represent the graph of this function?

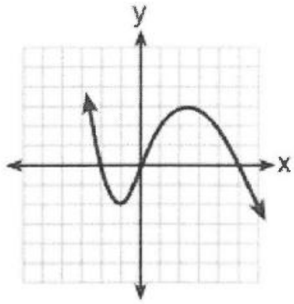
I



II



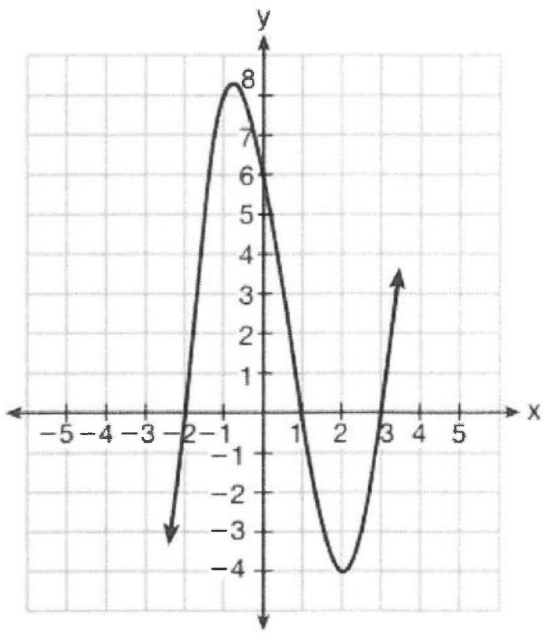
III



a) I, only
b) II, only
c) I and III
d) I, II, and III

2) Which equation(s) represent the graph?

I $y = (x + 2)(x^2 - 4x - 12)$
II $y = (x - 3)(x^2 + x - 2)$
III $y = (x - 1)(x^2 - 5x - 6)$



a) I, only
b) II, only
c) I and II
d) II and III

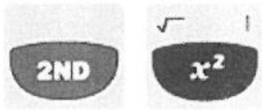
OBJECTIVE: SWBAT identify, describe, and graph special functions.

Graphing Square & Cube Roots on the Coordinate Plane:

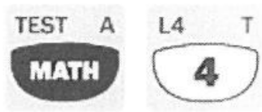
1) Press "Y="
2) Type in the function.
2) Go to the table.
3) List **at least 3** exact coordinates.
4) Plot the points.
5) Connect the points (look at the graph).

How to Type Roots:

For square root, press

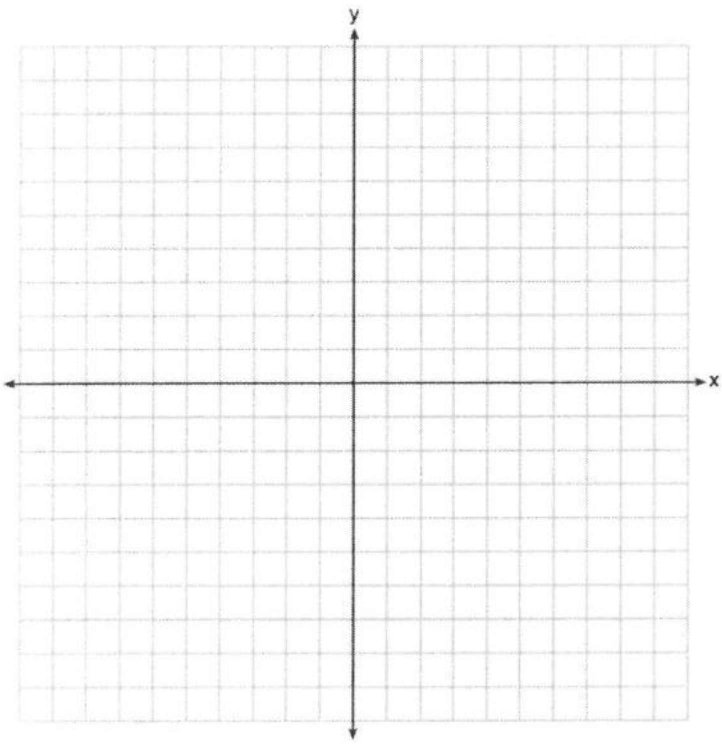


For cubed root, press



Practice:

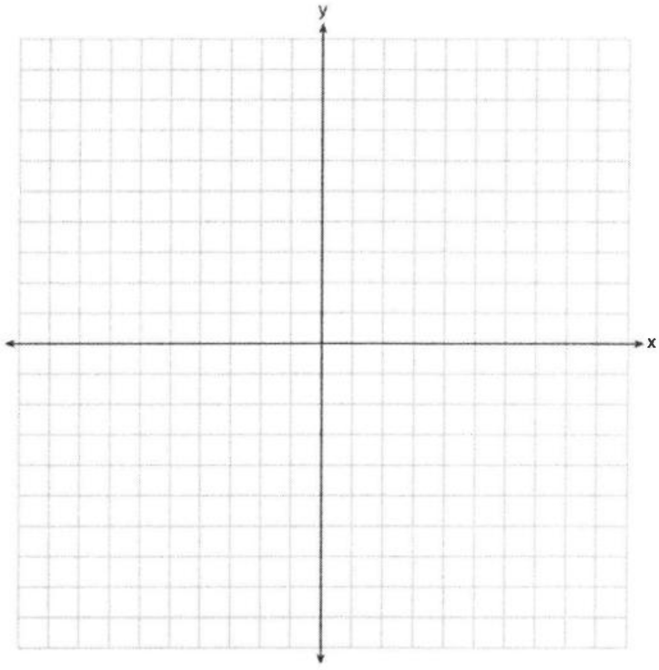
Create the graph of the following function: $y = \sqrt{x}$



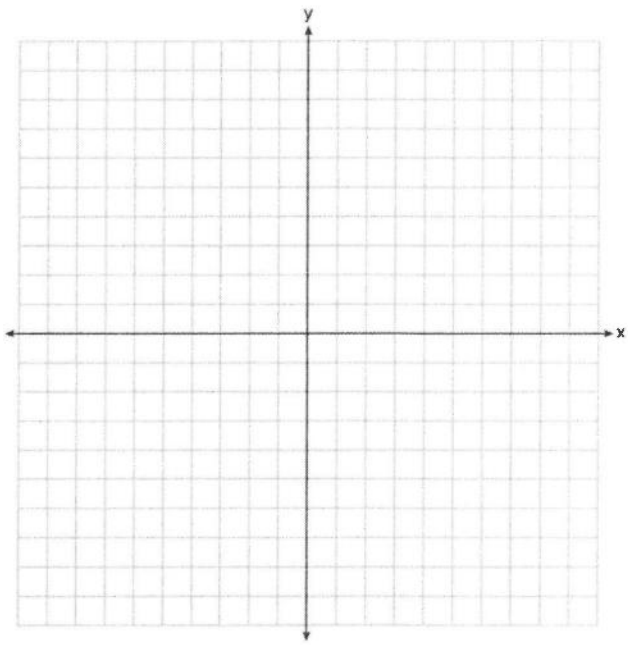
OBJECTIVE: SWBAT identify, describe, and graph special functions.

Practice:

Graph the function $y = -\sqrt{x+3}$ on the set of axes below.



Graph the function $y = \sqrt[3]{x}$ on the set of axes below.



OBJECTIVE: SWBAT identify, describe, and graph special functions.

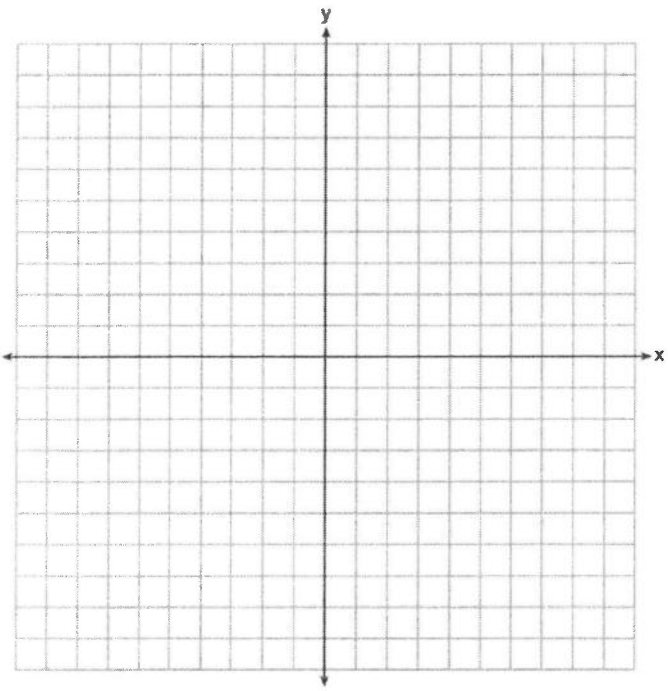
Absolute Value:

The absolute value gives us the length (distance) or magnitude of a number from 0. The absolute value of a number is always positive.

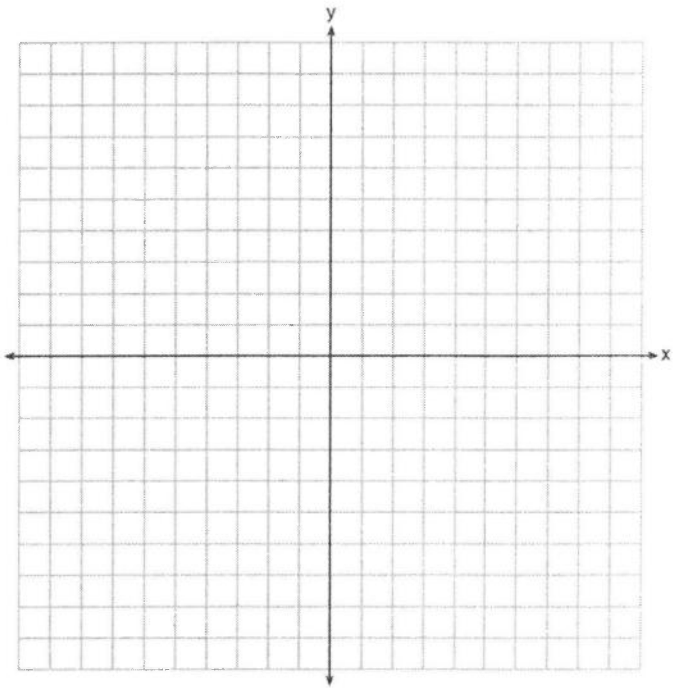
How to Graph an Absolute Value Function:

1) Press "Y ="
2) Press "Math."
3) Press the right arrow key →.
4) Press "Enter."
5) Type in the function. *You may need to close the absolute value with a parentheses.
6) Find the pattern.
7) Copy the table. You should have at least 5 coordinate pairs.
8) Plot the points.
9) Connect the points, label, and draw arrows if no domain is specified.

Create a table and graph the absolute value function: $y = |\,x - 3\,|$

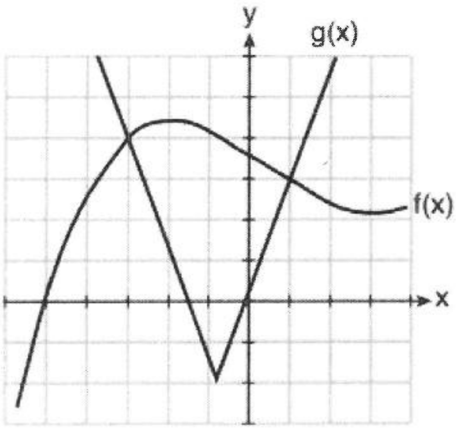
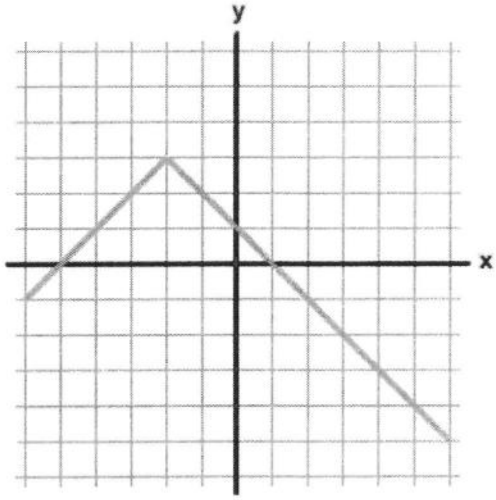


On the set of axes, graph the function: $y = |\,3x|$



OBJECTIVE: SWBAT identify, describe, and graph special functions.

Closing Assessment:

1) The graph below show two functions, $f(x)$ and $g(x)$. State all the values of x for which $f(x) = g(x)$? 	2) State the maximum of the function graphed below. 

Standards: CC.F.IF.7: Graphing Piecewise-Defined Functions: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Standards: F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Standards: F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

OBJECTIVE: SWBAT identify, create, and interpret the graph of a piecewise function.

Name: ________________________________ Date: ______________

Lesson 34: Piecewise Functions

Graphing Piecewise Functions:

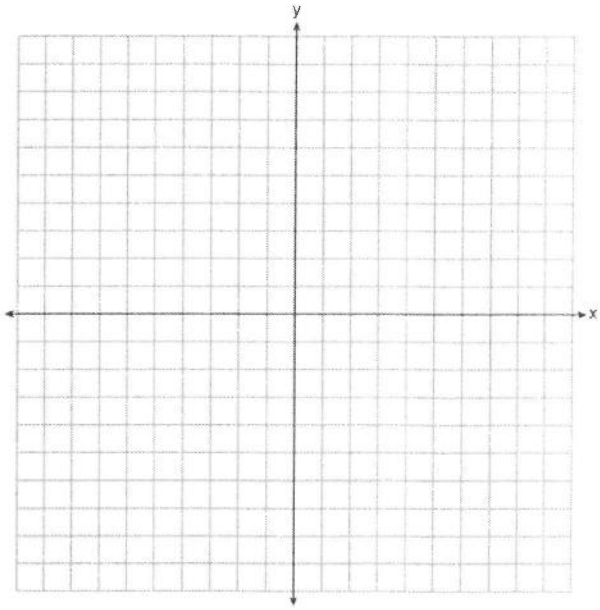
On the set of axes below, graph

$$g(x) = \frac{1}{2}x + 1$$

and

$$f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2 - x^2, & x > -1 \end{cases}$$

How many values of x satisfy the equation $f(x) = g(x)$? Explain your answer, using evidence from your graphs.

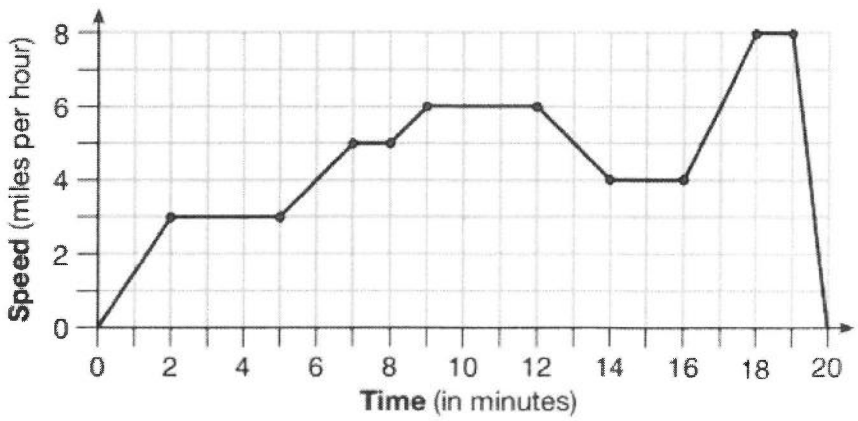


Practice:

1) The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.

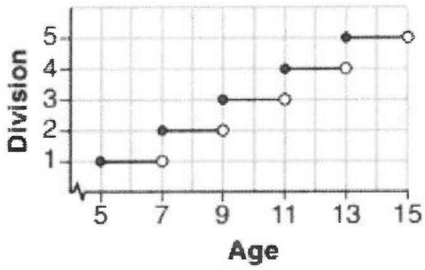
Which statement best describes what the jogger was doing during the 9-12 minute interval of her jog?

a) She was standing still.
b) She was increasing her speed.
c) She was decreasing her speed.
d) She was jogging at a constant rate.

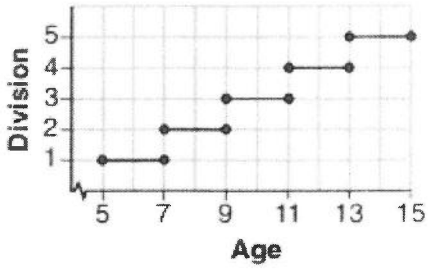


OBJECTIVE: SWBAT identify, create, and interpret the graph of a piecewise function.

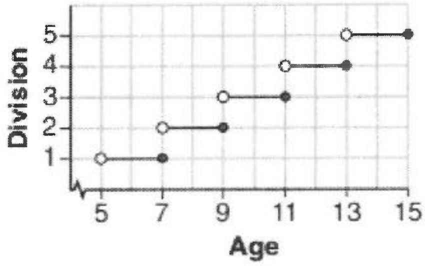
2) Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?



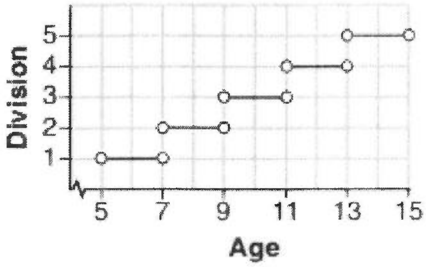
(1)



(3)

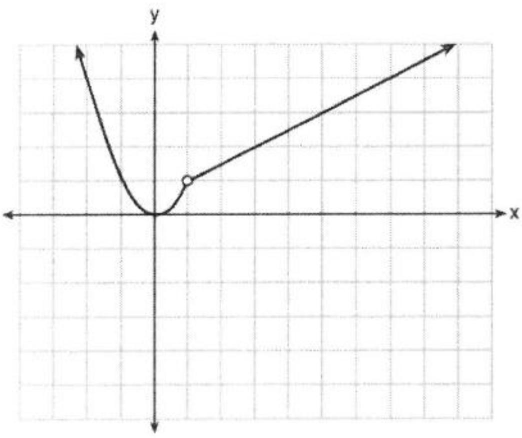


(2)



(4)

3) A function is graphed on the set of axes below.



Which function is related to the graph?

(a) $f(x) = \begin{cases} x^2, & x < 1 \\ x - 2, & x > 1 \end{cases}$

(b) $f(x) = \begin{cases} x^2, & x < 1 \\ 2x - 7, & x > 1 \end{cases}$

(c) $f(x) = \begin{cases} x^2, & x < 1 \\ \frac{1}{2}x + \frac{1}{2}, & x > 1 \end{cases}$

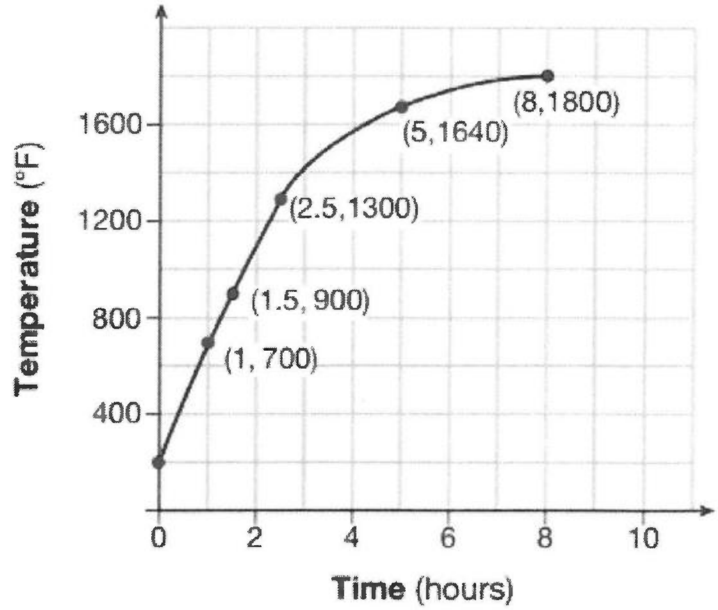
(d) $f(x) = \begin{cases} x^2, & x < 1 \\ \frac{3}{2}x - \frac{9}{2}, & x > 1 \end{cases}$

OBJECTIVE: SWBAT identify, create, and interpret the graph of a piecewise function.

4) Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph below shows the temperatures in a kiln while firing a piece of pottery after the kiln is preheated to 200°F.

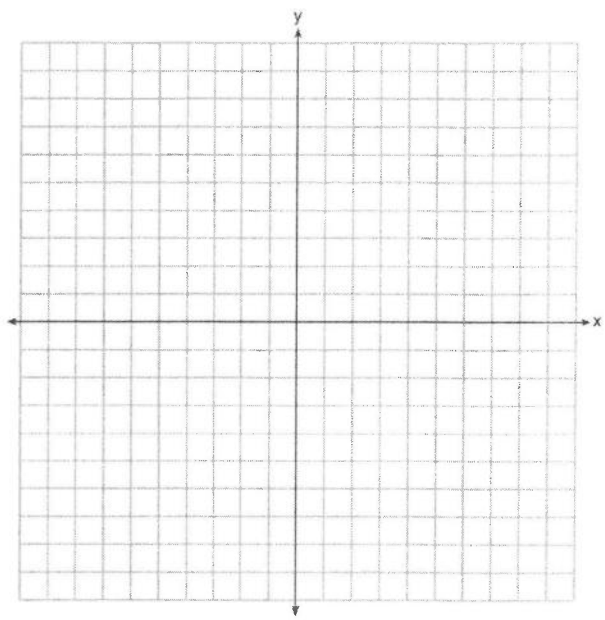
During which time interval did the temperature in the kiln show the greatest average rate of change?

a) 0 to 1 hour
b) 1 hour to 1.5 hours
c) 2.5 hours to 5 hours
d) 5 hours to 8 hours



5) Graph the following function on the set of axes below.

$$f(x) = \begin{cases} |x|, & -3 \le x < 1 \\ 4, & 1 \le x \le 8 \end{cases}$$



OBJECTIVE: SWBAT identify, create, and interpret the graph of a piecewise function.

6) The equation to determine the weekly earnings of an employee at The Hamburger Shack is given by $w(x)$, where x is the number of hours worked.

$$w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$$

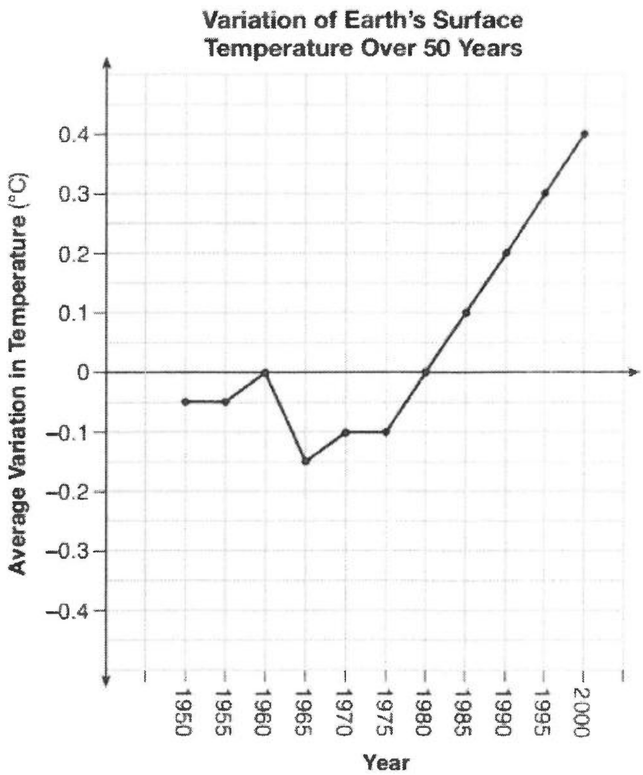
Determine the difference in salary, in dollars, for an employee who works 52 hours versus one who works 38 hours.

Determine the number of hours an employee must work in order to earn \$445. Explain how you arrived at this answer.

Closing Assessment:

The graph below shows the variation in the average temperature of Earth's surface from 1950-2000, according to one source.

During which years did the temperature variation change the most per unit time? Explain how you determined your answer.



Standards: F-IF.C.7 Graphing Piecewise-Defined Functions: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

OBJECTIVE: SWBAT create the graph of a system of functions.

Name: ______________________________ Date: ______________

Lesson 35: Graphing Systems

Systems with Absolute Value Functions:

1) The graphs of the functions $f(x) = |x-3| + 1$ and $g(x) = 2x + 1$ are drawn. Which statement about these functions is true?

a) The solution to $f(x) = g(x)$ is 3.

b) The solution to $f(x) = g(x)$ is 1.

c) The graphs intersect when $y = 1$.

d) The graphs intersect when $x = 3$.

2) Two functions, $y = |x-3|$ and $3x + 3y = 27$, are graphed on the same set of axes. Which statement is true about the solution to the system of equations?

a) (3, 0) is the solution to the system because it satisfies the equation $y = |x-3|$.

b) (9, 0) is the solution to the system because it satisfies the equation $3x + 3y = 27$.

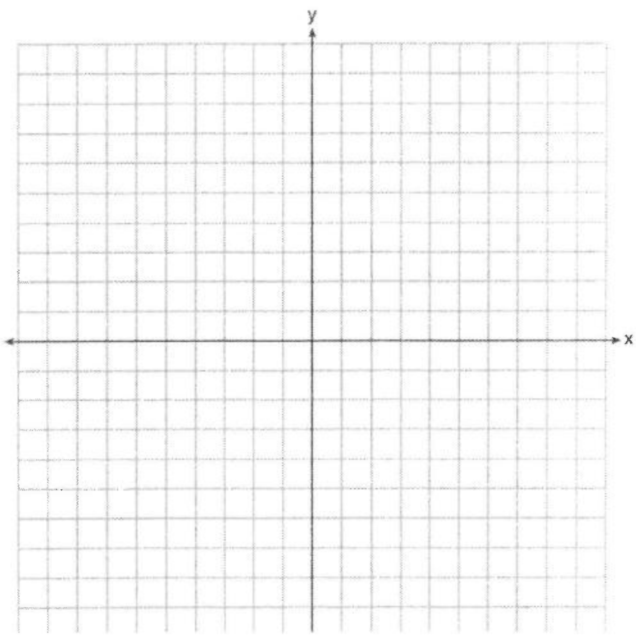
c) (6, 3) is the solution to the system because it satisfies both equations.

d) (3, 0), (9, 0), and (6, 3) are the solutions to the system of equations because they all satisfy at least one of the equations.

3) Graph $f(x) = |x|$ and $g(x) = -x^2 + 6$ on the grid.

Does $f(-2) = g(-2)$?

Use your graph to explain why or why not.

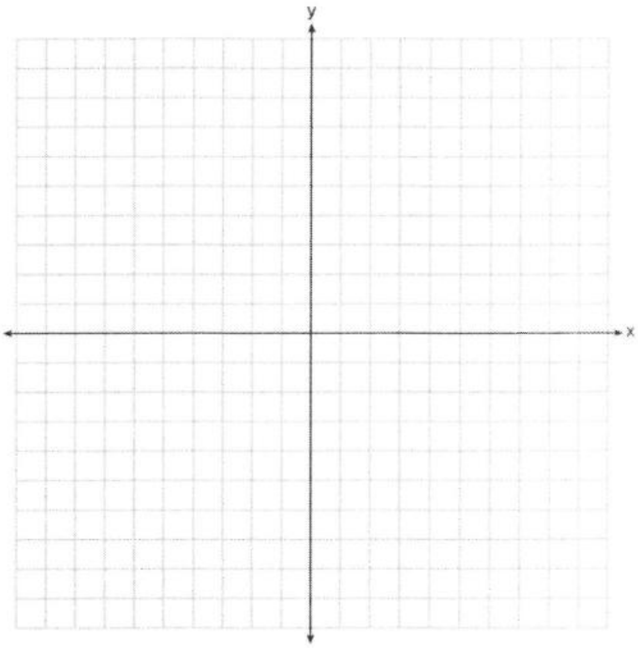


OBJECTIVE: SWBAT create the graph of a system of functions.

Linear-Exponential System:

Create the graph of the following system of equations:

$$y = \begin{cases} 8\left(\frac{1}{2}\right)^x \\ -2x + 1 \end{cases}$$



How To:

1) Graph the linear function.
2) Press "Y="
3) Type in the exponential function.
4) Go to the table.
5) List **at least 3** exact coordinates.
6) Plot the points.
7) Connect the points (refer to the graph).

Linear-Quadratic Systems:

We can solve a system of equations with one linear function and one quadratic function graphically.

How To Solve a Quadratic-Linear System Graphically:

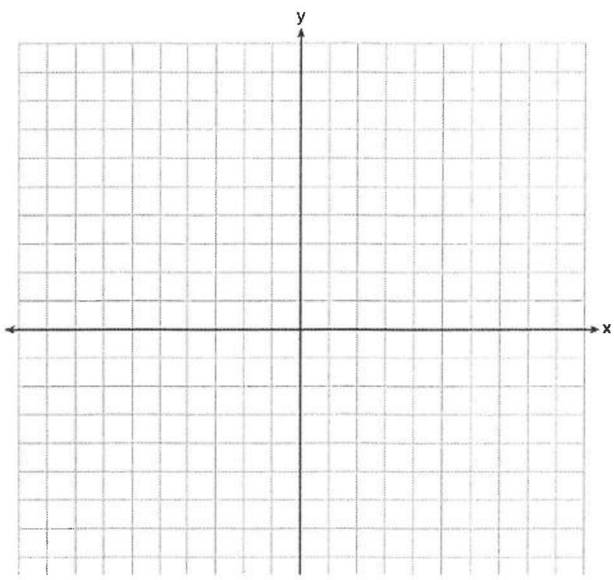
1. Graph the parabola: use the calculator to make a table of at least 5 coordinate points.
2. Graph the line: $y = mx + b$. (Remember to identify the slope and y-intercept)
3. Label everything (axes, line, parabola, and points of intersection).
4. Check your solutuion in each equation algebracially.

*Remember: The solution(s) to a system of equations is where their graphs intersect.

$y = x^2 + 4x + 1$

$y = 5x + 3$

Solution(s): ____________________



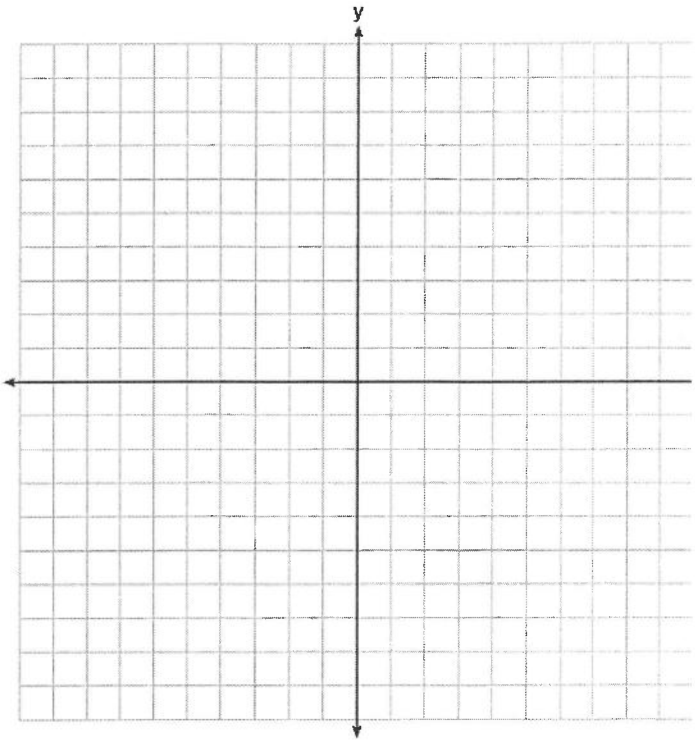
OBJECTIVE: SWBAT create the graph of a system of functions.

Practice:

1) Graph the following equations on the same set of axes.

$y = x - 8$

$y + 4 = x^2 - 3x$



Solution(s): ____________________

How many solutions does this system have?

How can you tell?

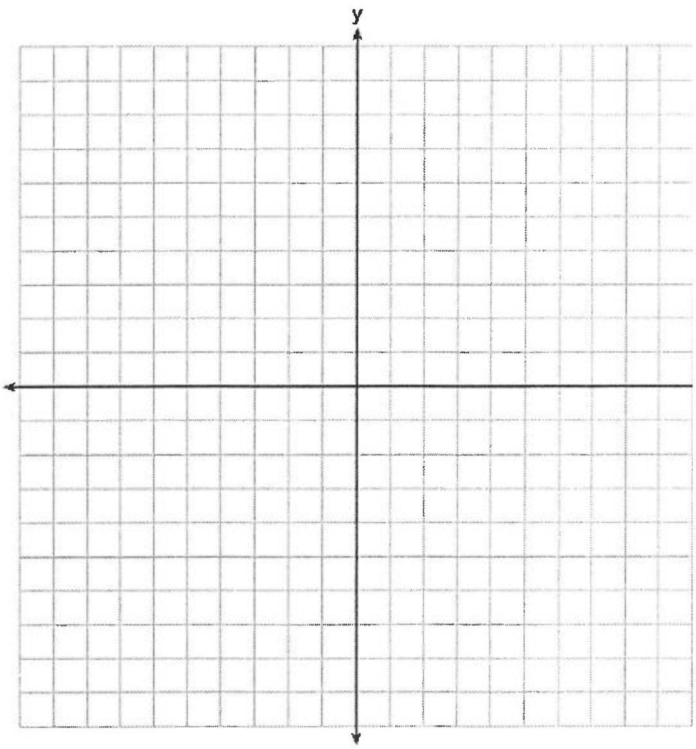
____

____

2) Graph the following equations on the same set of axes.

$y = x^2 - x - 6$

$y + 9 = x$



Solution(s): ____________________

How many solutions does this system have?

How can you tell?

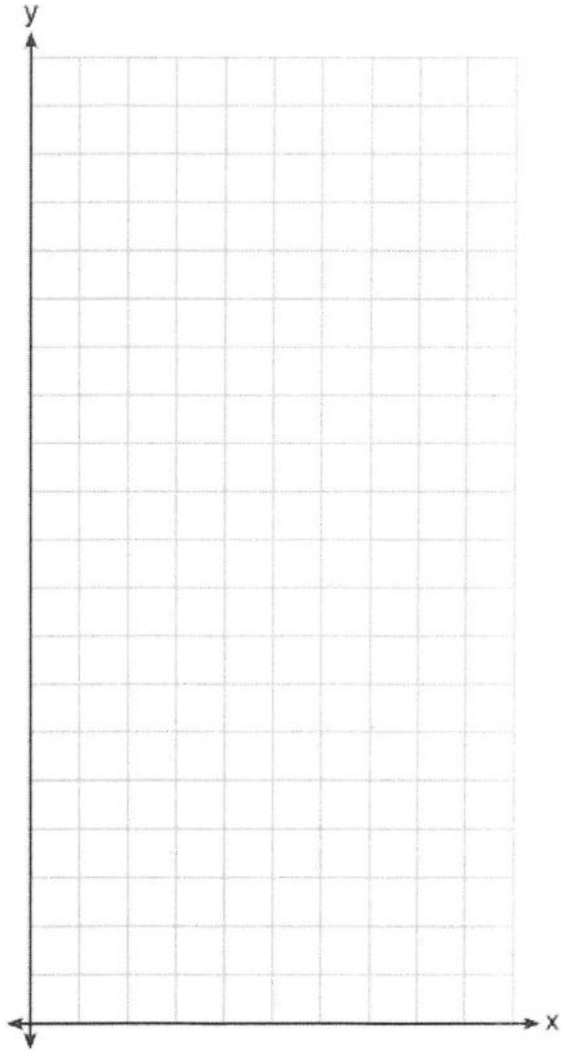
____

____

OBJECTIVE: SWBAT create the graph of a system of functions.

Quadratic-Exponential System

Graph $f(x) = x^2$ and $g(x) = 2^x$ for $x \geq 0$ on the set of axes below.

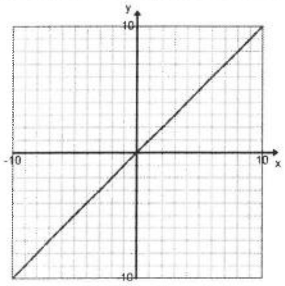
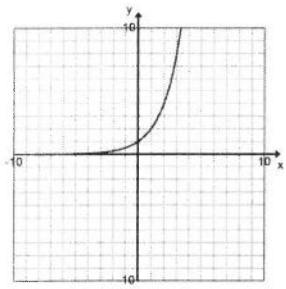
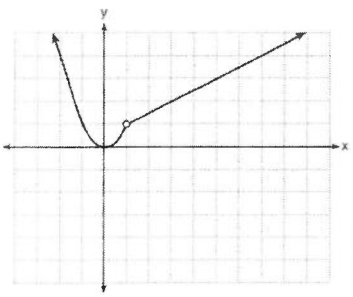


State which function, $f(x)$ or $g(x)$, has a greater value when $x = 20$. Justify your reasoning.

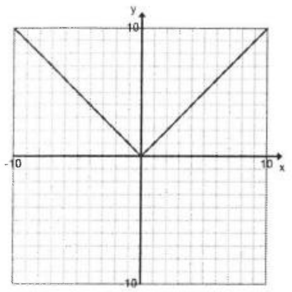
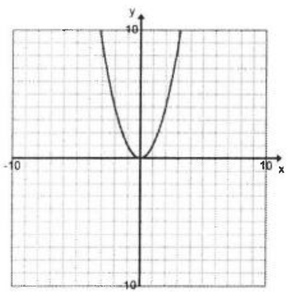
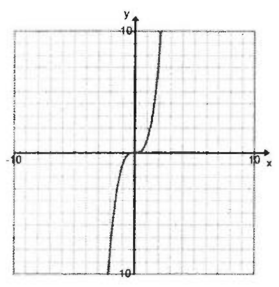
OBJECTIVE: SWBAT create the graph of a system of functions.

Comparing Functions:

Write the function name, the parent function equation, and a description of each type of function based on its graph.

Function Summary Chart			
Graph	**Function Name**	**Parent Function Equation**	**Description**
			
			
			

OBJECTIVE: SWBAT create the graph of a system of functions.

Closing Assessment:

1) What is the largest integer, x, for which the value of $f(x) = 5x^4 + 30x^2 + 9$ will be greater than the value of $g(x) = 3^x$? a) 7 b) 8 c) 9 d) 10	2) As x increases beyond 25, which function will have the largest value? a) $f(x) = 1.5^x$ b) $g(x) = 1.5x + 3$ c) $h(x) = 1.5x^2$ d) $k(x) = 1.5x^3 + 1.5x^2$

Standards: F-IF.C.7 Graphing Piecewise-Defined Functions. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative). F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Focus Standard: CC.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y=-3x and the circle x2+y2=3. CC.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

OBJECTIVE: SWBAT determine and describe the vertical and/or horizontal shift applied to a function in a graph and equation.

Name: ______________________________ Date: ______________

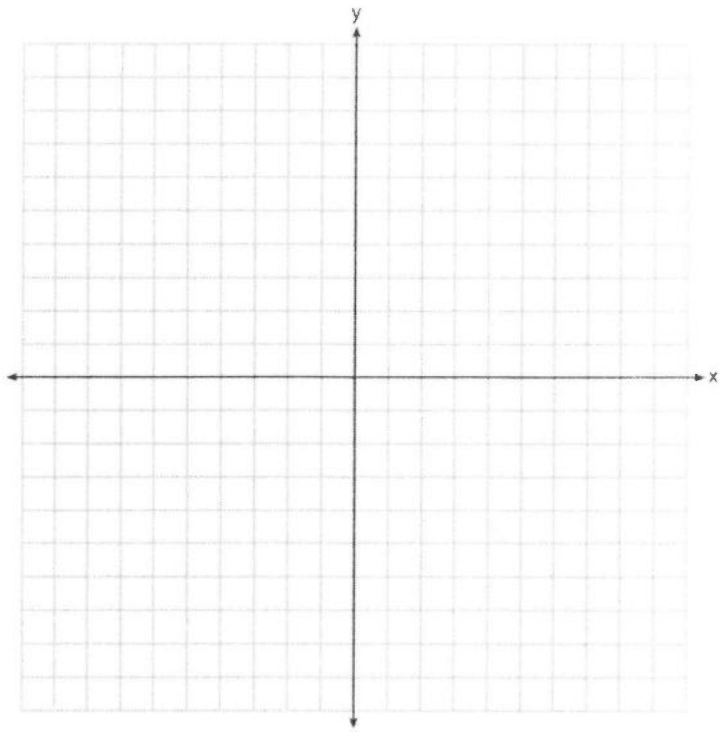
Lesson 36: Transformations

Vertical Shifts:

Graph the functions on the same graph.

$f(x) = x^2$ and $g(x) = x^2 + 5$

How does the graph of $g(x)$ compare to $f(x)$?



Vertical Shifting: The function $f(x) + k$ shifts the function k units up.

The function $f(x) - k$ shifts the function k units down.

How would the graph of $y = |x|$ be shifted in order to produce the graph of $y = |x| - 8$?

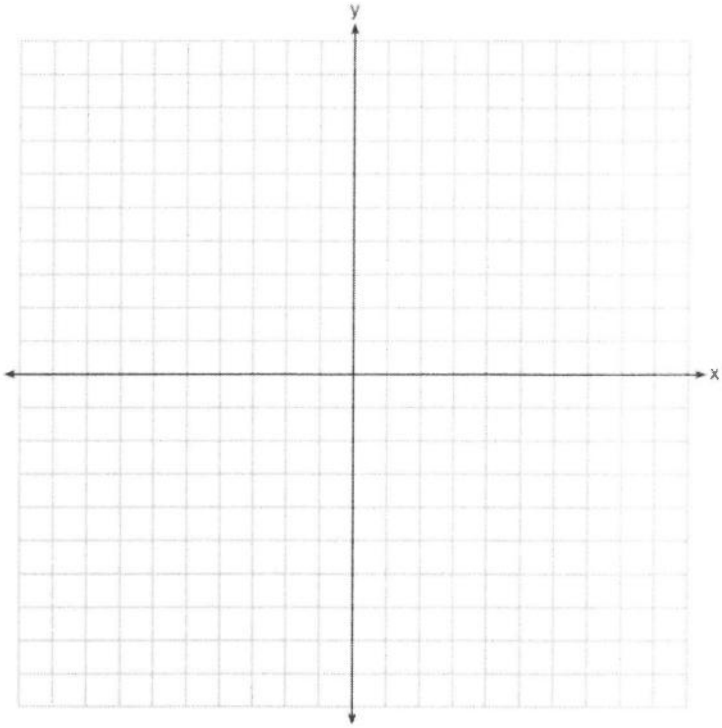
How would the graph of $h(x) = x^2$ be shifted in order to produce the graph of $d(x) = x^2 + 17$

OBJECTIVE: SWBAT determine and describe the vertical and/or horizontal shift applied to a function in a graph and equation.

Horizontal Shifts:

Graph the functions on the same graph.

$f(x) = x^2$ and $g(x) = (x + 5)^2$



How does the graph of $g(x)$ compare to $f(x)$?

Horizontal Shifting: The function $f(x + h)$ shifts the function h units left.

The function $f(x - h)$ shifts the function h units right.

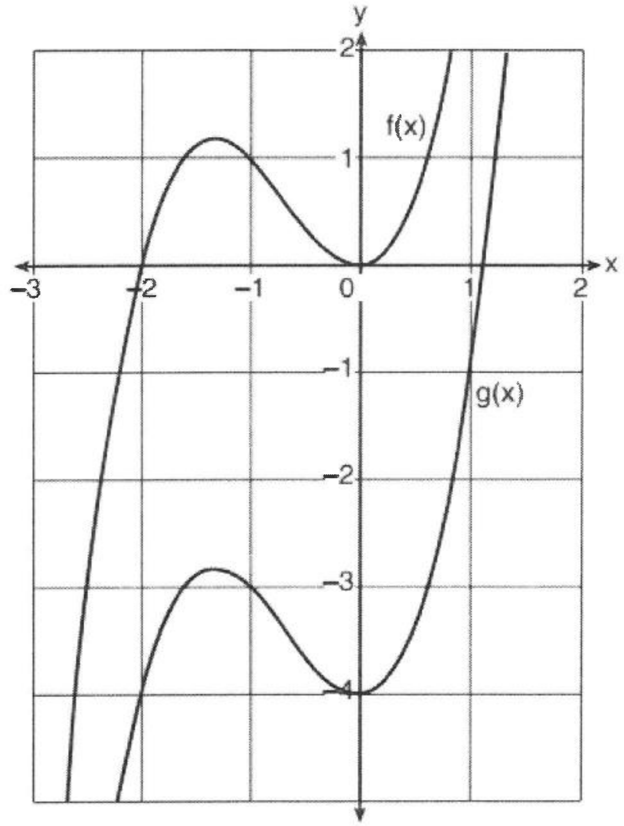
How would the graph of $y = |x|$ be shifted in order to produce the graph of $y = |x - 3|$?

How would the graph of $h(x) = x^2$ be shifted in order to produce the graph of $d(x) = (x + 6)^2$?

OBJECTIVE: SWBAT determine and describe the vertical and/or horizontal shift applied to a function in a graph and equation.

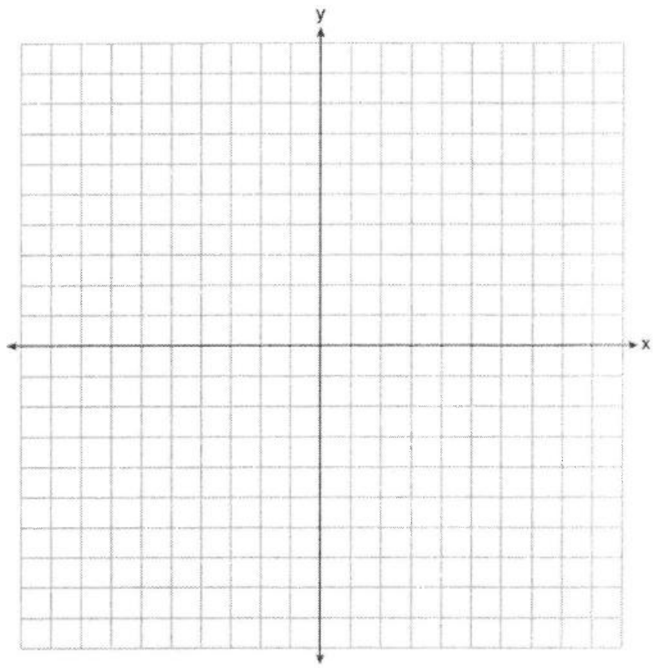
Practice:

1) In the diagram below, $f(x) = x^3 + 2x^2$ is graphed. Also graphed is $g(x)$, the result of a translation of $f(x)$.



Determine an equation of $g(x)$. Explain your reasoning.

2) On the axes, graph $f(x) = |3x|$.



If $g(x) = f(x) - 2$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?

If $h(x) = f(x - 4)$, how is the graph of $f(x)$ translated to form the graph of $h(x)$?

OBJECTIVE: SWBAT determine and describe the vertical and/or horizontal shift applied to a function in a graph and equation.

More Narrow/Wider Graphs

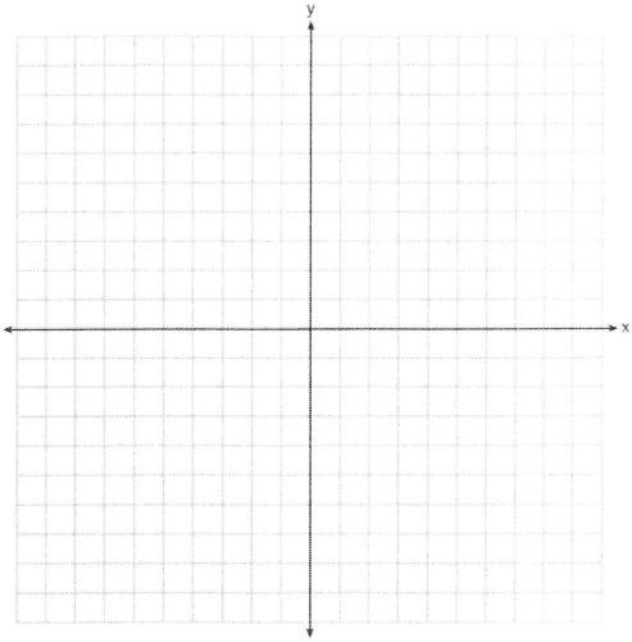
When $\boldsymbol{a} > \mathbf{1}$ (bigger than 1) the function $a \cdot f(x)$ makes the graph more narrow.

When $\mathbf{0} < \boldsymbol{a} < \mathbf{1}$, (fraction, decimal, etc.) the function $a \cdot f(x)$ makes the graph more wide.

Practice:

a) Graph the functions on the same graph.

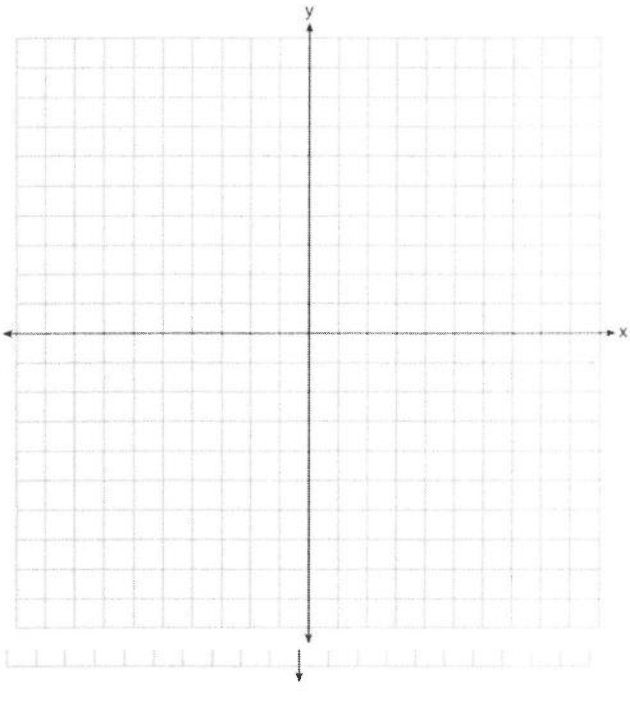
$f(x) = x^2$ and $g(x) = 2x^2$



How does the graph of $g(x)$ compare to $f(x)$?

b) Graph the functions on the same graph.

$f(x) = x^2$ and $h(x) = \frac{1}{2}x^2$



How does the graph of $h(x)$ compare to $f(x)$?

OBJECTIVE: SWBAT determine and describe the vertical and/or horizontal shift applied to a function in a graph and equation.

Practice:

1) Which statement best describes the change in the graph of $f(x) = x^2$ when the leading coefficient is multiplied by 4?

(1) The parabola becomes wider.

(2) The parabola becomes narrower.

(3) The parabola will shift up four units

(4) The parabola will shift right four units

2) How does the graph of $f(x) = 3(x-1)^2 + 7$ compare to the graph of $g(x) = x^2$?

(1) The graph of $f(x)$ is wider than the graph of $g(x)$, and its shifted left 2 units and up 7 units.

(2) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its shifted right 2 units and up 7 units.

(3) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its shifted left 2 units and up 7 units.

(4) The graph of $f(x)$ is wider than the graph of $g(x)$, and its shifted right 2 units and up 7 units.

3) Given: $f(x) = x^2$

a) If $g(x) = 3f(x)$, how is the graph of $f(x)$ transformed to form the graph of $g(x)$?

b) If $h(x) = \frac{1}{4}f(x)$, how is the graph of $f(x)$ transformed to form the graph of $h(x)$?

c) If $j(x) = 0.5f(x)$, how is the graph of $f(x)$ transformed to form the graph of $f(x)$?

OBJECTIVE: SWBAT determine and describe the vertical and/or horizontal shift applied to a function in a graph and equation.

Closing Assessment:

For each of the following, describe what happens to the graph for positive and negative values.

$f(x) + k$		$f(x + l)$		$mf(x)$	
Positive	Negative	Positive	Negative	Positive	Negative
				$m > 1$	
				$0 < m < 1$	

Standards: F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k.